This document is a preprint of: P. Rost and G. Fettweis, “Achievable Rates in Gaussian Half-Duplex Multiple Relay Networks,” in Proceedings of IEEE Global Communications Conference (GLOBECOM 2008), New Orleans, USA, Nov 2008. DOI:10.1109/GLOCOM.2008.ECP.239

Achievable Rates in Gaussian Half-Duplex Multiple Relay Networks

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***Abstract*— In this paper we analyze several protocols for Gaussian half-duplex multiple relay networks where each node either transmits or receives on a particular resource. We present achievable rates for a multilevel partial decode-and-forward approach which generalizes previous results presented by Kramer and Khojastepour *et al.*. Furthermore, we present a compress-and-forward protocol and a mixed strategy which use a regular encoding scheme. Finally, we discuss results for a four-terminal line network and compare the performance for fixed and random transmission strategies as well as for coherent and non-coherent transmission.**

I. Introduction

Next generation mobile communications systems are characterized by an increased density and availability of mobile terminals. To satisfy growing service demands and to exploit networks more efficiently, future infrastructure based wireless communications systems are likely to apply network coding techniques, in particular *relaying*. Relaying protocols utilize additional, intermediate nodes which support communication pairs; the principles of relaying were introduced in [1] and substantially refined for the three-terminal case in [2].

With the growing importance of mobile terminal networks, more recent work concentrates on relay networks of arbitrary size. In this context, general *decode-and-forward* (DF) and *compress-and-forward* (CF) strategies have been presented in [3]. The former one is an application of Slepian-Wolf coding, where each relay fully decodes the source message and provides additional, redundant information. By contrast, the compress-and-forward strategy is an application of Wyner-Ziv coding such that each relay quantizes its channel output which is decoded and exploited by the information sink.

Practical limitations imply an *orthogonality constraint* on relay nodes, i. e., relays either transmit or listen on a particular time-frequency resource. First analyses considering this constraint were presented in [4]–[6] for one relay and in [7] which derives outer bounds on the capacity of N-terminal networks. More recently, [8] presents a detailed analysis of CF and DF approaches regarding their diversity-multiplexing tradeoff. Furthermore, [9] presents strategies which exploit a random channel access to exchange information.

In [10] we applied the idea of [9] to the discrete memoryless multiple relay channel and presented different protocols and their achievable rates. In this paper, these protocols are applied to the Gaussian N-terminal channel to compare achievable rates as well as issues regarding their implementation in a mobile communications system. Section II defines the underlying system model. In Section III we derive the achievable rates for a partial DF protocol, before Section IV presents a CF approach with a regular encoding structure. Section V presents the achievable rates of a mixed protocol. Finally, we analyze numerical results for a four-terminal line-network in Section VI.

II. System model and notations

This paper uses non-italic uppercase letters X to denote complex Gaussian random variables (r.v.s) and italic letters (N or n) to denote constant values. Ordered sets are denoted by $\mathcal{X}$, their size is given by $|\cdot|$, the j-th element of a set is denoted by $\mathcal{X}(j)$, and $[b; b+k]$ represents the ordered set of numbers $(b, b+1, \cdots, b+k)$ or $\emptyset$ if $k < 0$. We use $\pi(\mathcal{X})$ to define the set of all permutations of $\mathcal{X}$. Let further N^k be a scalar parameterized using k then $N^{\mathcal{C}}$ denotes $\sum_{k\in\mathcal{C}} N^k$. Matrices are denoted by boldface uppercase letters $\mathbf{K}$, the determinant of a Matrix is denoted by $\|\cdot\|$ and the element in the i-th row and j-th column of matrix $\mathbf{K}$ is denoted by $[\mathbf{K}]_{i,j}$. Throughout this paper, we use $p(\mathrm{x}|\mathrm{y})$ to abbreviate the conditional probability density function (pdf) $p_{\mathrm{X}|\mathrm{Y}}(\mathrm{x}|\mathrm{y})$ for the benefit of readability. The mutual information function $\mathrm{C}(x)$ is defined as $\mathrm{C}(x) = \mathrm{ld}(1 + x)$.

In this paper we consider a Gaussian multiple relay channel consisting of $N+2$ nodes: the source node $s = 0$, the set of N relays $\mathcal{R} := [1; N]$ and the destination $d = N + 1$. We define the node states $(\mathrm{m}_s, \mathrm{m}_1, \ldots, \mathrm{m}_N) \in \mathcal{M}_s \times \mathcal{M}_1 \times \cdots \mathcal{M}_N$ with $\mathcal{M}_t = \{L, T\}$. Each $t \in [0; N]$ is either listening ($\mathrm{M}_t = L$) or transmitting ($\mathrm{M}_t = T$) on a particular resource. It is possible that the source remains silent, e. g., to reduce interference in a wireless network. The source chooses an ordering of all relay nodes $o \in \pi([1; N + 1])$ where $o(N + 1) = N + 1$. For the benefit of readability we abbreviate in the following $\mathrm{Y}_{o(l)}$ by Y_l and the relay $o(l)$ by relay l. All results presented in the sequel are given for a specific o, though a maximization over $\pi([1; N + 1])$ is necessary.

Let $d_{l',l}$ be the distance between nodes l' and $l \neq l'$ and θ the pathloss exponent, then the gain factor between both nodes is given in a log-distance path loss model by $g_{l',l} = d_{l',l}^{-\theta/2}$. The channel input at node l is given by the n-length sequence of complex Gaussian r.v.s $\{\mathrm{X}_l[i]\}_{i=1}^n$ with zero mean and variance P_l, denoted by $\mathrm{X}_l \overset{n}{\sim} \mathcal{CN}(0, P_l)$. From the orthogonality constraint follows $(\mathrm{M}_l[i] = L) \rightarrow (\mathrm{X}_l[i] = 0)$, which implies that each node $l \in [0; N]$ must fulfill the power

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constraint $\frac{1}{n}\sum_{i=1}^{n} \mathrm{E}\left\{|\mathrm{X}_l[i]|^2\right\} = p_{\mathrm{M}_l}(T)\, P_l$. Let further the channel output at node l and time instance i be given by

$$\mathrm{Y}_l[i] = 1\left(\mathrm{M}_l[i] = L\right) \cdot \left(\sum_{l' \in [0;N] \setminus l} g_{l',l} \mathrm{X}_{l'}[i] + Z_l[i] \right),$$

where $1(\cdot)$ is the indicator function returning 1 if its argument is true and 0 otherwise, and $Z_l \overset{n}{\sim} \mathcal{CN}(0, N_l)$ is the additive white Gaussian noise at node l. As an immediate consequence of the orthogonality constraint we can state that $(\mathrm{M}_l[i] = T) \rightarrow (\mathrm{Y}_l[i] = 0)$. We further divide all transmissions in blocks $b \in [1;B]$ of length n and use the standard definition of achievable rates as given in [11].

III. Partial decode-and-forward

Our first proposal is an application of partial DF to a half-duplex relay network. Consider Fig. 1 which illustrates the structure of this protocol: The source message $\mathrm{W} \in [1; 2^{nR}]$ is mapped to $N+1$ message levels $\mathrm{U}_s^k \overset{n}{\sim} \mathcal{CN}(0,1)$, $k \in [1; N+1]$, with rates R_s^k. Each relay $l \in [1;N]$ creates the messages $\mathrm{U}_l^k \overset{n}{\sim} \mathcal{CN}(0,1)$, $k \in [1;l]$, with rates R_s^k.

Assume that relay $l \in [1;N]$ has successfully decoded the indices of the first l source message levels $\mathrm{U}_s^{[1;l]}$ transmitted in block b. In block $b+l$ this relay selects the relay messages $\mathrm{U}_l^{[1;l]}$ assigned to the same indices. Since each relay l delays the transmission of redundant information by l blocks, it knows the first l message levels of all successive relays. Hence, to exploit coherent transmission (if possible), it creates the relay transmission as $\mathrm{X}_l[i] = 1\left(\mathrm{M}_l[i] = T\right) \mathrm{X}'_l[i]$ with

$$\mathrm{X}'_l[i] = \sqrt{P_l} \sum_{k=1}^{l} \sum_{l'=l}^{N} \left(1\left(\mathrm{M}_{l'}[i] = T\right) \sqrt{\alpha_{l,l'}^k} \mathrm{U}_{l'}^k[i] \right).$$

Here, $\alpha_{l,l'}^k$ models both the power distribution for the own messages ($l = l'$) and the coherent support for all successive relays, i. e., $(l > l' \vee k > l) \rightarrow \alpha_{l,l'}^k = 0$. Similarly, due to the delay at each relay, the source has knowledge of each relay message (which are functions of the source message) and it creates its messages as $\mathrm{X}_s[i] = 1\left(\mathrm{M}_s[i] = T\right) \mathrm{X}'_s[i]$ with

$$\mathrm{X}'_s[i] = \sqrt{P_s} \sum_{k=1}^{N+1} \left[\sqrt{\alpha_{s,s}^k} \mathrm{U}_s^k + \sum_{l=k}^{N} 1\left(\mathrm{M}_l[i] = T\right) \sqrt{\alpha_{s,l}^k} \mathrm{U}_l^k \right].$$

For the sake of readability, these definitions choose $\alpha_{l,l'}^k$ independent of the current states $\mathrm{M}_{[1;N]}$ though an extension using state adaptive $\alpha_{l,l'}^k$ is possible.

Finally, consider the decoding at node l at the end of block $b+l-1$. From the previous description we know that in blocks $[b; b+l-1]$ nodes $[0; l-1]$ transmit information for the source message of block b. Due to the regular encoding structure, i. e., each node transmits the message with the same index, node l tries to find a unique message index for the tuple $\left(\mathrm{U}_s^k, \mathrm{U}_k^k, \ldots, \mathrm{U}_{l-1}^k\right)$ such that all messages are jointly typical. As already mentioned, node l additionally knows all message levels $[1;l]$ transmitted by nodes $l' > l$ and the indices for the source message levels $[1; k-1]$ (as we assume that all message levels are decoded in ascending order).

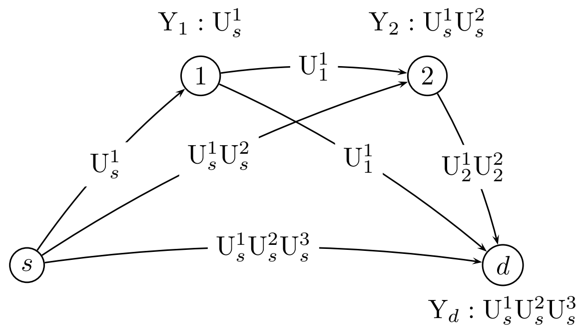

Fig. 1. *Information exchange* of partial DF for $N = 2$.

At node l the received power for message U_j^k is given for a particular state realization $\mathrm{m}_{[0;N]}$ by

$$\Gamma_{j,l}^k\left(\mathrm{m}_{[0;N]}\right) = \left| \sum_{j' \in \{s, [k;j]\}} 1\left(\mathrm{m}_{j'} = T\right) \cdot \left(g_{j',l} \sqrt{\alpha_{j',j}^k P_t} \right) \right|^2.$$

With our specific decoding order the received power at node l before decoding U_j^k is given by $\sigma_{j,l}^k\left(\mathrm{m}_{[0;N]}\right)$:

$$\begin{aligned}
\sigma_{s,l}^k\left(\mathrm{m}_{[0;N]}\right) &= \Gamma_{s,l}^{[k;N+1]}\left(\mathrm{m}_{[0;N]}\right) + \Gamma_{j\in[l+1;N],l}^{[l+1;j]}\left(\mathrm{m}_{[0;N]}\right) + N_l \\
\sigma_{j,l}^k\left(\mathrm{m}_{[0;N]}\right) &= \Gamma_{s,l}^{[1;N+1]}\left(\mathrm{m}_{[0;N]}\right) + \Gamma_{j'\in[1;j-1],l}^{[1;j']}\left(\mathrm{m}_{[0;N]}\right) \\
&\quad + \Gamma_{j,l}^{[k;j]}\left(\mathrm{m}_{[0;N]}\right) + \Gamma_{j'\in[l+1;N],l}^{[l+1;j']}\left(\mathrm{m}_{[0;N]}\right) + N_l,
\end{aligned}$$

where $\Gamma_{l'\in\mathcal{T},l}^{[1;l']}\left(\mathrm{m}_{[0;N]}\right) = \sum_{l'\in\mathcal{T}} \Gamma_{l',l}^{[1;l']}\left(\mathrm{m}_{[0;N]}\right)$ and $\Gamma_{j,l}^k$ represents the useful power within $\sigma_{j,l}^k$. Due to the random channel access of nodes $[0;N]$, we can give the pdf of Y_l after a polar coordinate transformation as

$$p_{j,l}^k\left(\mathrm{y}, \mathrm{m}_{\mathcal{L}}\right) = \sum_{\mathrm{m}_{\overline{\mathcal{L}}} \in \mathcal{M}_{\overline{\mathcal{L}}}} p\left(\mathrm{m}_{\overline{\mathcal{L}}} | \mathrm{m}_{\mathcal{L}}\right) \frac{\exp\left(-\frac{\mathrm{y}}{\sigma_{j,l}^k\left(\mathrm{m}_{[0;N]}\right)} \right)}{\pi \sigma_{j,l}^k\left(\mathrm{m}_{[0;N]}\right)}, \tag{1}$$

where the states of nodes $\mathcal{L}$ are known and those of nodes $\overline{\mathcal{L}} = [0;N] \setminus \{\mathcal{L}, l\}$ are unknown. Furthermore, its differential entropy is defined as [11, Definition 9.1]

$$\mathrm{h}_{j,l}^k\left(\mathrm{m}_{\mathcal{L}}\right) = -\pi \int_0^\infty p_{j,l}^k\left(\mathrm{y}, \mathrm{m}_{\mathcal{L}}\right) \mathrm{ld}\left(p_{j,l}^k\left(\mathrm{y}, \mathrm{m}_{\mathcal{L}}\right)\right) \mathrm{dy}. \tag{2}$$

Using the previous definitions and protocol description we can state the following theorem on the achievable rates.

Theorem 1: The achievable rates $R = \sum_{k=1}^{N+1} R_s^k$ for the previously described partial DF protocol must satisfy

$$R_s^1 \leq \sup_{p,\alpha} \min_{l\in[1;N+1]} Q_{s,l}^1\left(\mathcal{L}_1\right) + \sum_{j=1}^{l-1} Q_{j,l}^1\left(\mathcal{L}_{j+1}\right) \tag{3}$$

$$R_s^k \leq \sup_{p,\alpha} \min_{l\in[k;N+1]} Q_{s,l}^k\left(\mathcal{L}_0\right) + \sum_{j=k}^{l-1} Q_{j,l}^k\left(\mathcal{L}_j\right) \tag{4}$$

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with $\mathcal{L}_j = [j;N] \setminus \{l\}$ and

$$\mathrm{Q}^1_{j,l}(\mathcal{L}) = \sum_{\mathrm{m}_\mathcal{L}\in\mathcal{M}_\mathcal{L}} p(\mathrm{m}_l = L|\mathrm{m}_\mathcal{L}) \cdot p(\mathrm{m}_\mathcal{L}) \Big(\mathrm{h}^1_{j,l}(\mathrm{m}_\mathcal{L}) - \sum_{\mathrm{m}_j\in\mathcal{M}_j} p\left(\mathrm{m}_j|\mathrm{m}_{\{\mathcal{L},l\}}\right) \mathrm{h}^2_{j,l}\left(\mathrm{m}_{\{\mathcal{L},j\}}\right)\Big), \quad (5)$$

$$\mathrm{Q}^k_{j,l}(\mathcal{L}) = \sum_{\mathrm{m}_\mathcal{L}\in\mathcal{M}_\mathcal{L}} p(\mathrm{m}_l = L|\mathrm{m}_\mathcal{L}) \cdot p(\mathrm{m}_\mathcal{L}) \cdot \left(\mathrm{h}^k_{j,l}(\mathrm{m}_\mathcal{L}) - \mathrm{h}^{k+1}_{j,l}(\mathrm{m}_\mathcal{L})\right), \quad (6)$$

where $\mathrm{h}^k_{j,l}(\mathrm{m}_\mathcal{L})$ is given in (2) and the supremum in (3) and (4) is taken over all $\alpha^k_{l,l}$ and the joint pdf $p_{\mathrm{M}_{[0;N]}}$.

Proof: The proof follows a strict application of [10, Theorem 1] to the previously defined Gaussian system model. ∎

Theorem 1 and in particular equations (5) and (6) reflect the random channel access and as a result the improved rates. Nonetheless, one can see the difficulty to evaluate the previous integrals as $p^k_{j,l'}$ is the weighted sum of exponential pdfs. The more general integral

$$\int_0^\infty \left(\sum_k \frac{a_k\lambda_k}{\pi} e^{-\lambda_k y}\right) \mathrm{ld}\left(\sum_k \frac{a_k\lambda_k}{\pi} e^{-\lambda_k y}\right) \mathrm{d}y$$

can only be loosely upper and lower bounded (using log-sum inequality and Jensen's inequality). Hence, we must evaluate this term numerically as long as $\overline{\mathcal{L}} \neq \emptyset$ in (1). If we use only one message level, i. e., $\alpha^k_{l,l'} = 0$ for $k > 1$, the previous theorem yields the results given in [9]. Furthermore, if we manipulate $p_{\mathrm{M}_{[0;N]}}$ such that it satisfies

$$\forall l \in [0;N] : \Pr\left(\mathrm{m}_l = T \middle| \exists j \in [1;k-1] : \mathrm{m}_{l-j} = T\right) = 0,$$

we have the multihop protocol with limited resource reuse $1/k$ described in [12].

IV. A COMPRESS-AND-FORWARD APPROACH

Next, we present a compress-and-forward approach where, in contrast to the previous proposal, the relays neither need to decode the source nor other relay messages. Actually, as Section VI reveals, due to the orthogonality constraint the requirement to decode at the relays is a severe bottleneck in the considered system. In comparison to the previous section we now assume an arbitrary transmission schedule which is known to all nodes, i. e. a fixed transmission schedule. There are two reasons for this choice: first, due to the nature of CF, node $l \in [0;N-1]$ is unable to anticipate what any node $l' > l$ is transmitting and hence no coherent transmission is possible compared to partial DF. Furthermore, the fixed schedule can improve the interference mitigation between individual nodes in wireless networks.

Each relay $l \in [1;N]$ creates $2^{n\Delta_l}$ quantization messages $\hat{\mathrm{Y}}_l$ which are used to quantize its channel output. The introduced quantization noise is modeled at each node by $\hat{N}_l$. Besides, each relay creates $2^{n\Delta_l}$ messages X_l which are used to communicate the quantized channel output. Consider some block $b \in [1;B]$: the relay l searches for a quantization with index $q_{l,b+1}$ which is jointly typical with its channel output $\mathrm{y}_l(b)$. In block $b+1$ it transmits the broadcast message X_l assigned to the same index.

On the other hand, in block b the destination decodes the source message transmitted in block $b-N$. At first, it decodes the quantization of relay N by searching for the set of all possible broadcast messages X_N in block b and the set of all possible quantizations $\hat{\mathrm{Y}}_l$ for block $b-1$ such that exactly one message index is element of both sets. Obviously, this quantization can be used to support the decoding of the broadcast message X_{N-1} transmitted in block $b-1$. Hence, using a recursive decoding process the destination can finally use the quantizations of all relay nodes and its own channel output to decode the source message.

Due to the usage of a regular encoding structure, i. e., we generate the same number of quantization and broadcast messages, no intermediate mapping of quantization to broadcast messages is necessary and the encoding as well as decoding process is simplified. In this example, the achievable rates are not affected but the next section will present a mixed strategy where the rates are improved through this regular encoding.

From the previous description we have $\alpha^k_{j,j'} = 0$ if $k > 1 \vee j \neq j'$, i. e., only one message level and no coherent transmission. Using our previously defined system model the received power at node l from the set of nodes $\mathcal{L}'$ for a particular realization $\mathrm{m}_{[0;N]} \in \mathcal{M}^{N+1}$ is given by $\Gamma^1_{\mathcal{L}',l}\left(\mathrm{m}_{[0;N]}\right)$. The covariance of the channel outputs at nodes l and l' is given by $\tilde{\Gamma}^1_{\mathcal{L}',l,l'}\left(\mathrm{m}_{[0;N]}\right)$. In the following, we will omit the superscripts for the sake of readability. Now define the set $\mathcal{L}$ with $\mathcal{L}\cap\mathcal{L}' = \emptyset$. Furthermore, let the set of all nodes in $\mathcal{L}$ which are currently listening be

$$\mathcal{T}_{\mathcal{L},\mathrm{m}_{[0;N]}} = \{l \in \mathcal{L} : \mathrm{M}_l = L\}.$$

Then the covariance matrix $\mathbf{K}_{\mathcal{L},\mathcal{L}'}\left(\mathrm{m}_{[0;N]}\right)$ of all quantizations at nodes $l \in \mathcal{T}_{\mathcal{L},\mathrm{m}_{[0;N]}}$ and Y_d is given by (in the following definition we abbreviate $\mathcal{T}_{\mathcal{L},\mathrm{m}_{[0;N]}}$ by $\mathcal{T}$):

$$\begin{aligned}
\left[\mathbf{K}_{\mathcal{L},\mathcal{L}'}\left(\mathrm{m}_{[0;N]}\right)\right]_{1,1} &= \Gamma^1_{\mathcal{L}',d} + N_d \\
\left[\mathbf{K}_{\mathcal{L},\mathcal{L}'}\left(\mathrm{m}_{[0;N]}\right)\right]_{j+1,j+1} &= \Gamma^1_{\mathcal{L}',\mathcal{T}(j)} + \hat{N}_{\mathcal{T}(j)} + N_{\mathcal{T}(j)} \\
\left[\mathbf{K}_{\mathcal{L},\mathcal{L}'}\left(\mathrm{m}_{[0;N]}\right)\right]_{1,j+1} &= \tilde{\Gamma}^1_{\mathcal{L}',d,\mathcal{T}(j)} \\
\left[\mathbf{K}_{\mathcal{L},\mathcal{L}'}\left(\mathrm{m}_{[0;N]}\right)\right]_{j'+1,j+1} &= \tilde{\Gamma}^1_{\mathcal{L}',\mathcal{T}(j'),\mathcal{T}(j)}
\end{aligned}$$

with $j \in \left[1;\left|\mathcal{T}_{\mathcal{L},\mathrm{m}_{[0;N]}}\right|\right]$. For the benefit of readability the arguments of $\Gamma^1_{\mathcal{L}',l}$ and $\tilde{\Gamma}^1_{\mathcal{L}',l,l'}$ are omited. With these definitions we can now formulate the following theorem.

Theorem 2: With a fixed schedule the presented compress-and-forward approach achieves any rates satisfying

$$R \leq \sum_{\mathrm{m}_{[0;N]}\in\mathcal{M}_{[0;N]}} p\left(\mathrm{m}_{[0;N]}\right) \mathrm{ld}\left(\frac{\left\|\mathbf{K}_{[1;N],s}\left(\mathrm{m}_{[0;N]}\right)\right\|}{\left\|\mathbf{K}_{[1;N],\emptyset}\left(\mathrm{m}_{[0;N]}\right)\right\|}\right), \quad (7)$$

subject to (8) on the top of the next page.

Proof: The proof follows a strict application of [10, Theorem 2] to the previously defined system model using

$$\sum_{\left\{\mathrm{m}_{[0;N]}\in\mathcal{M}_{[0;N]}:\mathrm{m}_{N-l}=L\right\}} p\left(\mathrm{m}_{[0;N]}\right)\left[\mathrm{C}\left(\frac{\Gamma_{[N-l;N],N-l}}{\Gamma_{[0;N-l-1],N-l}+\hat{N}_{N-l}+N_{N-l}}\right)+\mathrm{ld}\left(\frac{\left\|\mathbf{K}_{[N-l;N],[0;N-l-1]}\right\|}{\hat{N}_{N-l}\left\|\mathbf{K}_{[N-l+1;N],[0;N-l-1]}\right\|}\right)\right]$$
$$\leq \sum_{\left\{\mathrm{m}_{[0;N]}\in\mathcal{M}_{[0;N]}:\mathrm{m}_{N-l}=T\right\}} p_{\mathrm{M}_{[0;N]}}\left(\mathrm{m}_{[0;N]}\right)\mathrm{ld}\left(\frac{\left\|\mathbf{K}_{[N-l+1;N],[0;N-l]}\right\|}{\left\|\mathbf{K}_{[N-l+1;N],[0;N-l-1]}\right\|}\right). \quad (8)$$

the definition of differential entropy for multivariate complex Gaussian r.v.s [11, Ch. 9 and 10]. ■

Eq. (8) reflects the side condition on the quantization quality. The right hand side of (8) gives the channel coding constraint whereas the left hand side gives the source coding constraint. This inequality can not be solved in general and hence a numerical optimization is necessary. Furthermore, due to its structure, (8) must be solved iteratively in descending order, starting with $\hat{N}_N$. Compared to the CF approach presented in [3, Theorem 3], our approach does not include any interference cancellation at the relay nodes itself but only at the destination. Besides, we utilize a regular encoding approach which simplifies the encoding and decoding structure as well as improves the rates in case of multiple descriptors.

V. A MIXED PROTOCOL FOR TWO RELAYS

In the previous sections we presented approaches where *all* relays operate *either* in decode-and-forward *or* compress-and-forward mode. Considering a mobile communications system with fixed infrastructure relays this imposes rather strict constraints on the deployment of relays. In case all relays are operating in DF mode, they should be placed such that a sufficiently good connection between relay and source exists. On the other hand, if CF relays are used we must assure that each relay has a high-quality link towards the destination. Relays might switch between DF and CF in up- and downlink but we are likely to face situations where neither strategy is optimal.

In Section VI we analyze a line network where two alternately transmitting relays are able to achieve capacity in our region of interest. In the following we present a protocol using two alternately transmitting relays where one relay operates in DF mode and the other one in CF mode. The idea of using alternately transmitting relays was introduced in [13] and since then received strong attention, e. g., the Diamond network in [14] and an analysis of different DF and CF based protocols in [15]. Our proposal does not exploit all possible degrees of freedom such as multilevel encoding at the source or a full characterization of the multiple description problem at the CF relay. However, we use a regular CF approach which improves the achievable rates.

The structure of the protocol is illustrated in Fig. 2. Each transmission block is divided into two phases with probabilities p_1 and p_2 where we require that during the first phase only relay 1 and during phase 2 only relay 2 transmits. The source divides its transmission into two independent data streams

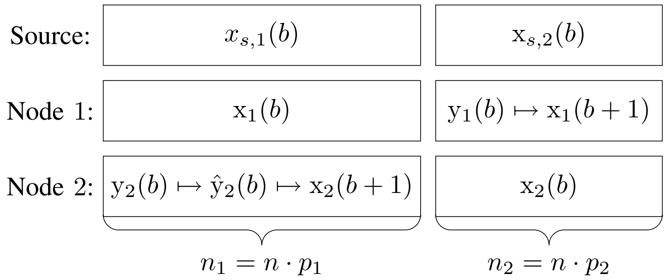

Fig. 2. Coding structure for the mixed strategy with two alternately transmitting relay nodes.

with $2^{nR_{\mathrm{CF}}}$ messages $\mathrm{X}_{s,1}\overset{n_1}{\sim}\mathcal{CN}(0,1)$ and $2^{nR_{\mathrm{DF}}}$ messages $\mathrm{X}_{s,2}\overset{n_2}{\sim}\mathcal{CN}(0,1)$ where $n_1=p_1n$ and $n_2=p_2n$. Relay 2 generates $2^{n\Delta_2}$ quantizations $\hat{\mathrm{Y}}_2\overset{n_1}{\sim}\mathcal{CN}\left(0,\sigma^2_{\mathrm{Y}_2}+\hat{N}_2\right)$ where $\sigma^2_{\mathrm{Y}_2}$ denotes the variance of its channel output and $\hat{N}_2$ models the quantization noise. Besides it generates $2^{n\Delta_2}$ broadcast messages $\mathrm{X}_2\overset{n_2}{\sim}\mathcal{CN}(0,1)$. Relay 1 generates $2^{nR_{\mathrm{DF}}}$ messages $\mathrm{X}_1\overset{n_1}{\sim}\mathcal{CN}(0,1)$. The channel input and output at each node is defined as in the previous two sections. We further introduce an adaptive $\alpha_{s,1}$ such that $(\mathrm{M}_1=L)\rightarrow\alpha_{s,1}=0$. This is useful as the source does not need to support the first relay ($\alpha_{s,1}>0$) if relay 1 is receiving. Besides, note that $\alpha_{s,2}=0$ as a coherent transmission is not possible for CF cooperation.

Consider again Fig. 2: in phase 1 the source transmits the CF part $\mathrm{X}_{s,1}$. It is received by relay 2 which quantizes its channel output using $\hat{\mathrm{Y}}_2$. The resulting index gives the broadcast message transmitted in phase 2 of the next block. In phase 2 the source transmits the DF part which is decoded by relay 1. In phase 1 of the next block the relay transmits its message associated to the same source message index. Before the relay is decoding the source message it might decode the other relay's transmission. For this purpose it can exploit the knowledge that $\hat{\mathrm{Y}}_2$ depends on the transmission of relay 1. Furthermore, the relay can exploit its own channel output Y_1 as side information. Alternatively, the relay can ignore the transmission of relay 2 and treat it as interference. This might be better modeled using a multilevel encoding approach at relay 2 but as already mentioned we want to keep the protocol as simple as possible.

At the end of block b the destination decodes at first the quantization of relay 2 for block $b-1$. It can exploit its own channel output and the index transmitted by relay 2 in block b. Using this quantization and the own channel output in block $b-1$ the destination builds the set of all possible indices transmitted by relay 1 in block $b-1$. Using the quantization for block $b-2$ and its own channel output the destination further builds the set of all possible source message indices transmitted in block $b-2$. By building the intersection of both sets the destination decodes $\mathrm{X}_{s,2}$ transmitted in phase 2 of block $b-2$. Finally, knowing the transmission of relay 1 in block $b-2$ (decoded in block $b-1$) it uses the quantization of relay 2 (decoded in block $b-1$) and its own channel output to decode $\mathrm{X}_{s,1}$ transmitted in block $b-2$.

To formulate the achievable rates of this protocol, we define the covariance matrices $\mathbf{K}_1 = \mathbf{K}_{2,\{s,1\}}\left(\mathrm{M}_{[1,2]} = \{T, L\}\right)$ of the quantized channel output at relay 2 and the channel output at the destination before the transmission of relay 1 is decoded and $\mathbf{K}_2 = \mathbf{K}_{2,\{s\}}\left(\mathrm{M}_{[1,2]} = \{T, L\}\right)$ after this transmission is decoded. Using both covariance matrices we are able to formulate the following theorem.

Theorem 3: The previously presented mixed protocol achieves any rate $R = R_{\mathrm{DF}} + R_{\mathrm{CF}}$ satisfying

$$R_{\mathrm{DF}} \leq \min\left\{p_2 \mathrm{C}\left(\frac{\Gamma_{s,d}}{N_d}\right) + p_1 \mathrm{ld}\left(\frac{\|\mathbf{K}_1\|}{\|\mathbf{K}_2\|}\right), p_2 \mathrm{C}\left(\frac{\Gamma_{s,1}}{N_1}\right)\right\}, \tag{8}$$

if node 1 decodes the quantization of node 2, and

$$R_{\mathrm{DF}} \leq \min\left\{p_2 \mathrm{C}\left(\frac{\Gamma_{s,d}}{N_d}\right) + p_1 \mathrm{ld}\left(\frac{\|\mathbf{K}_1\|}{\|\mathbf{K}_2\|}\right), p_2 \mathrm{C}\left(\frac{\Gamma_{s,1}}{N_1 + \Gamma_{2,1}}\right)\right\}, \tag{9}$$

otherwise. The compress-and-forward rate is limited by

$$R_{\mathrm{CF}} \leq p_1 \mathrm{C}\left(\frac{\Gamma_{s,2}}{N_2 + \hat{N}_2} + \frac{\Gamma_{s,d}}{N_d}\right), \tag{10}$$

subject to

$$\hat{N}_2 \geq \frac{\Gamma_{\{s,1\},2} + N_2 - \frac{\left(\tilde{\Gamma}_{\{s,1\},2,d}\right)^2}{\Gamma_{\{s,1\},d} + N_d}}{2^{\hat{R}_2^1/p_1} - 1} \tag{11}$$

with

$$\hat{R}_2^1 = p_2 \mathrm{C}\left(\frac{\Gamma_{2,d}}{N_d + \Gamma_{s,d}}\right),$$

and if relay 1 decodes $\hat{\mathrm{Y}}_2$ also subject to

$$\hat{N}_2 \geq (\Gamma_{s,2} + N_2) \cdot \left(2^{\hat{R}_2^2/p_1} - 1\right)^{-1} \tag{12}$$

with

$$\hat{R}_2^2 = p_2 \mathrm{C}\left(\frac{\Gamma_{2,1}}{N_1 + \Gamma_{s,1}}\right).$$

Proof: The proof follows a strict application of [10, Theorem 3] to the previously defined system model. We again omit the indices of $\Gamma_{\mathcal{L}',l}$ and $\tilde{\Gamma}_{\mathcal{L}',l,l'}$. ∎

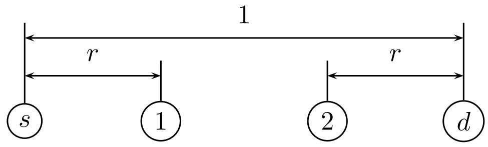

Fig. 3. Setup for our analysis

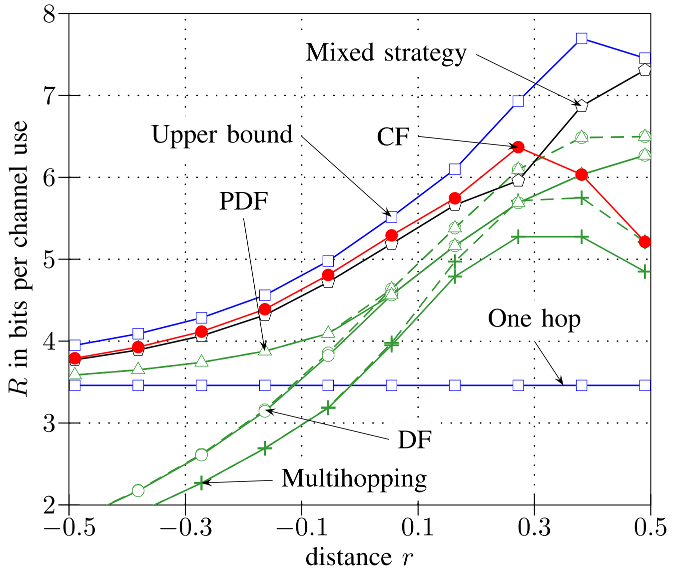

Fig. 4. Results for $\theta = 4$ and non-coherent transmission. Solid lines indicate results for fixed transmission schedules and dashed for random transmission schedules.

VI. Numerical Results

We analyze in this section the previously presented protocols using a line network consisting of two relays. More specifically, we investigate the achievable rates of partial decode-and-forward (PDF), decode-and-forward with one message level (DF), compress-and-forward (CF), DF with limited resource reuse (Multihopping), the mixed strategy presented in Section V and the capacity upper bound derived in [7]. a max-flow min-cut theorem.

A. Setup

Consider the setup illustrated in Fig. 3 where two relays are placed in one line with source and destination. In this setup all distances are normalized to the source-destination distance, i. e., $d_{s,d} = 1$, $d_{s,1} = d_{2,d} = |r|$ and $d_{1,2} = 1 - 2r$. This model should reflect different deployments which might be used in a mobile communications systems with fixed relay nodes. Although the situation $r < 0$ might happen (for instance due to shadowing), our region of interest is $0 < r < 0.5$ which coincides with a deployment where relays are placed between source and destination.

Let us further define $P_s = P_1 = P_2$, $N_1 = N_2 = N_d$ and $P_s/N_d = 10\,$dB. Different values for P_t and N_t would evidently impact the following quantitative statements, but the qualitative statements remain almost unaffected. We analyze the achievable rates for the Gaussian system model with a

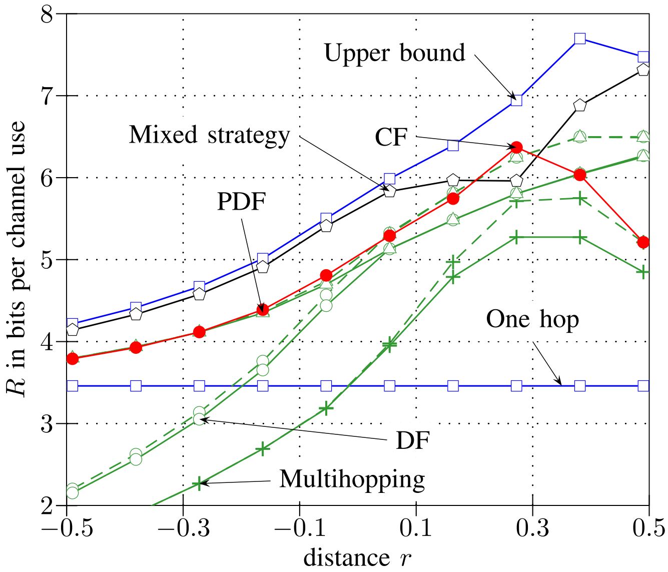

Fig. 5. Results for $\theta = 4$ and coherent transmission. Solid lines indicate results for fixed transmission schedules and dashed for random transmission schedules.

pathloss exponent $\theta = 4$ and compare the results for non-coherent transmission in Fig. 4 (e. g., for the case of phase-fading) and coherent transmission in Fig. 5.

B. Achievable rates

We can observe from both figures that a random transmission schedule provides only a minor performance benefit. More specifically, for $r < 0$ there is almost no benefit and for $r > 0$ the benefit is not more than 1 Bit.

We can further observe that decode-and-forward based protocols achieve a poor performance compared to full-duplex networks where DF based approaches achieve results near capacity [16]. The reason is that each relay must decode the complete or parts of the source message which significantly limits the achievable rates. Even the usage of partial decode-and-forward (PDF) does not improve the performance for $r > 0$ in comparison to decode-and-forward with one message level (DF) since the increased interference at each node outweighs the advantages obtained by using multiple encoding levels. For $r < 0$, PDF achieves better results than DF since relay 2 is turned off. Besides, compress-and-forward achieves reasonable results for $r < 0$ and non-coherent transmission but not for coherent transmission as compress-and-forward does not utilize any coherent support.

The best results are offered by the mixed strategy which achieves results close to capacity for $r < 0$ and $r \approx 0.5$. Its maximum performance advantage is provided for $r \approx 0.5$ where rates are improved by almost 4 Bits in comparison to direct transmission (one hop). Nonetheless, for $r \approx 1/3$ we have the worst-case situation for our mixed strategy. In this case the signal-to-noise ratios for the link from source to relay 1 and between both relays are similar. Hence, if relay 1 ignores the transmission of relay 2, the DF part achieves significantly lower rates. On the other hand, if it decodes the quantization of relay 2 the quantization quality is reduced due to the low inter-relay link quality.

C. Final remarks

The derived rate expressions are a useful framework to plan the deployment of relays in fixed infrastructure mobile communications systems as they provide all means to compute the achievable rates. We can observe from the previous analysis that the mixed strategy probably provides the best performance with low implementation complexity since no multilevel-encoding is used and each phase can operate independently. Besides, in a mobile communications system we will most likely have $r \approx 0$ or $r \approx 0.5$. The former one relates to the situation where each relay has a suitable link either to the source or the destination. The latter one corresponds to a deployment where relays are placed to form colocated virtual antenna arrays. Furthermore, the protocol can be used for both up- and downlink in comparison to DF and CF which are either recommendable in up- or downlink.

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