

A Reduced Complexity Time-Domain Transmitter for UF-OFDM

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Abstract—Upcoming fifth generation (5G) cellular networks will demand more from the physical layer (PHY) than current-generation Orthogonal Frequency Division Multiplexing (OFDM) can deliver. The 5G waveform candidate Universal Filtered OFDM (UF-OFDM) is designed to provide the flexibility required for future applications. However, the introduction of subband filters in UFMC can increase implementation complexity and low-complexity solutions need to be found. State-of-the-art technologies provide an algorithm that performs shorter-length FFTs that can reduce complexity to two to ten times that of OFDM (depending on the allocation sizes), at the cost of only approximating the exact UFMC signal. In this paper we propose a new approximation of the UFMC signal which bases on the similarity of adjacent subcarriers that can be implemented with reduced number of operations. Analysis show that the system can be implemented with only 20% more operations than standard OFDM when accepting some increase in the subband bandwidth. A more accurate solution can be implemented at roughly 3.6 times OFDM complexity. The results can reduce implementation costs for future mobile devices.

I. INTRODUCTION

Nowadays, OFDM is widely applied in a variety of wireless communication standards such as wireless Local Area Network (WLAN), Worldwide Interoperability for Microwave Access (WiMax) and digital video broadcasting terrestrial (DVB-T). Also, the downlink of the current fourth generation (4G) cellular standard Long-Term Evolution (LTE) employs OFDM on the PHY. OFDM offers the strengths of robustness against frequency-selective fading and a very simple transmitter and receiver implementation.

However, the upcoming 5G of cellular networks will demand more from the PHY that can be served by conventional OFDM. For example, upcoming systems should offer a reduced out-of-band (OOB) emission such that heterogeneous systems can asynchronously coexist next to each other. Clearly, OFDM can hardly address this requirement with its inherent high sidelobes due to the implicit usage of rectangular pulse shaping filters.

Instead, more advanced waveforms are being researched for upcoming 5G networks [1]. Filterbank multicarrier (FBMC) [2], Generalized Frequency Division Multiplexing (GFDM) [3] and Bi-orthogonal frequency division multiplexing (BFDM) [4] are promising candidates as they all offer a

significantly reduced OOB emission compared to conventional OFDM.

Another promising waveform candidate for 5G is UF-OFDM [5] (also known as Universal Filtered Multicarrier (UFMC)), which has improved spectral localization and offers more robustness against time-synchronization errors [6] compared to OFDM. UF-OFDM is based on an OFDM signal which is divided into subbands. Each subband is separately filtered to provide a low OOB emission. Since groups of subcarriers are jointly filtered, the bandwidth of the filters can be wider and accordingly the tails can be much shorter compared to per-subcarrier filtering as is done in e.g. FBMC [7]. Hence, UF-OFDM is more suitable for e.g. short-burst communication or low-latency constrained communication and allows using different parallel multi-carrier numerologies [8]. Replacing the cyclic prefix (CP) by a soft-symbol transition offers improved robustness against time-frequency misalignments [6], [9] and hence reduces requirement of an accurate synchronization as it is required for OFDM systems. This property helps to save signaling overhead, can increase battery life of portable devices and reduces oscillator requirements, leading to cheaper end-user equipment.

The complexity for the UF-OFDM receiver is in the order of two times that of OFDM of the same size, since the UF-OFDM receiver is simply performing a discrete Fourier transform (DFT) of twice the block size in order to take the subband filter tails into consideration. At the transmitter side, besides a description using classic matrix-multiplication [5], an approximate solution that generates the transmit signal in the frequency domain before transforming all subbands to the time domain is published in [10]. As the proposal suggests to modulate each subband separately, the computational complexity increases with the number of allocated subbands, up to approximately 10 times of the OFDM complexity for a full allocation in a system similar to LTE [10]. Since the solution scales linearly with the number of subbands, this is in particular disadvantageous for base stations, which usually require a high number of allocated subbands. Instead, a system that offers low complexity also for utilization of large fractions of the carrier band would be beneficial.

In this paper, we address this particular problem by proposing a time-domain implementation of the UF-OFDM trans-

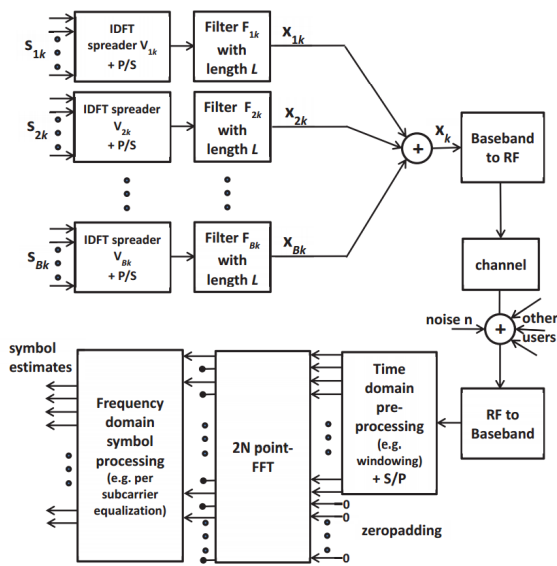


Fig. 1. Generic UF-OFDM baseline transceiver.

mitter. The proposal bases on the fact that the effective filters for groups of subcarriers can be approximated by a single filter which can be applied to all subcarriers in the group. Expressing the convolution by the filter as a time-domain windowing operation can, in this case, reduce the complexity at the cost of sacrificing an exact UF-OFDM signal generation. However, the main properties such as the low OOB emission of the signal are kept during the processing and the number of chosen subcarrier groups controls the accuracy. Using three subcarrier groups, the proposal achieves a load-independent complexity that is about 3.6 times that of OFDM at a negligible deviation from the exact UF-OFDM signal.

The remainder of this paper is organized as follows. Section II introduces the conventional UF-OFDM transmitter model and formulates the reduced complexity solution. The according complexity expressions are derived in Section III and Section IV provides numerical comparison of the results. Finally, Section V concludes the paper.

II. SYSTEM MODEL

In this section, the generic UF-OFDM transmitter is formulated. Subsequently, the proposed reduced-complexity solution is derived.

A. Baseline Transmitter

Fig. 1 shows a block diagram of the UF-OFDM transceiver. The UF-OFDM transmit signal consists of N subcarriers and is divided into K subbands containing Q subcarriers each. Note that $KQ < N$ is explicitly allowed and neither K nor Q need to divide N . In UF-OFDM, each subband is filtered by a subband filter \vec{f}_k that is derived by shifting the impulse response \vec{f} of length L of the prototype subband filter to the appropriate subband frequency. Then, the transmit signal of

one UF-OFDM block can be written in matrix-vector form as

$$\vec{x} = \sum_{k=0}^{K-1} \mathbf{F}_k \mathbf{V}_k \mathbf{N} \vec{s}_k, \quad (1)$$

where \vec{s}_k denotes the data to be transmitted on the k th subband. \mathbf{V}_k is a submatrix of the unitary N -point Inverse Discrete Fourier Transform (IDFT) matrix, that contains only the columns that correspond to the subcarrier frequencies of the k th subband. \mathbf{F}_k denotes the Toeplitz matrix of dimension $\tilde{N} \times N$ that convolves its argument with \vec{f}_k and $\tilde{N} = N + L - 1$. \mathbf{N}_k is an optional diagonal matrix that normalizes the transmit data such that each subcarrier is transmitted using the same energy. In this case, we refer to \vec{x} as the normalized UF-OFDM signal. If \mathbf{N} is the identity matrix, we refer to \vec{x} as the non-normalized UF-OFDM signal. In what follows, we omit \mathbf{N} and assume that the normalization is already performed on the transmit data. Eq. (1) can be formulated as a more explicit modulation equation, given by

$$x[n] = \sum_{k=0}^{K-1} f_k[n] * \left\{ \sum_{q=0}^{Q-1} s_{k,q} \exp\left(j2\pi \frac{K_0 + kQ + qn}{N}\right) R_N[n] \right\}, \quad (2)$$

where $n = 0, \dots, \tilde{N} - 1$, $*$ denotes linear convolution, K_0 denotes the starting frequency of the lowest subband and $R_N[n]$ denotes a rectangular window ranging from $n = 0, \dots, N - 1$. In (2), $s_{k,q}$ denotes the data symbol to be transmitted in the q th subcarrier of the k th subband. Given that $f_k[n]$ is given by shifting $f[n]$ to the center of the k th subband by

$$f_k[n] = \underbrace{f[n] \cdot \exp\left(j2\pi \frac{1/2Q}{N} n\right)}_{f_Q[n]} \cdot \exp\left(j2\pi \frac{K_0 + kQ}{N} n\right), \quad (3)$$

where $f_Q[n]$ is the filter prototype filter shifted by half the subband bandwidth. (2) can be reformulated to

$$x[n] = \sum_{q=0}^{Q-1} \sum_{k=0}^{K-1} s_{k,q} \left\{ \underbrace{(f_Q[n] * \exp(j2\pi \frac{qn}{N}))}_{f_q[n]} R_N[n] \cdot \exp\left(j2\pi \frac{K_0 + kQ}{N} n\right) \right\} \quad (4)$$

$$= \sum_{q=0}^{Q-1} \sum_{k=0}^{K-1} s_{k,q} f_q[n] \cdot \exp\left(j2\pi \frac{K_0 + kQ}{N} n\right). \quad (5)$$

Note that $f_q[n]$ can be understood as the effective filter that is used to modulate the q th subcarrier in each subband.

B. Reduced Complexity Formulation

To reduce the complexity of the transmitter, consider

$$f_q[n] = f_Q[n] * R_N[n] \exp\left(j2\pi \frac{qn}{N}\right) \quad (6)$$

$$F_q(f) = \text{DTFT}\{f_q[n]\} = F(f - \frac{q/2}{N}) \cdot D_N(f - q/N) \quad (7)$$

where $F(f) = \text{DTFT}\{f[n]\}$, $D_N(f) = \text{DTFT}\{R_N[n]\}$ is the n th Dirichlet kernel given by

$$D_N(f) = \frac{\sin(\pi N f)}{2\pi N \sin(f/2)} \quad (8)$$

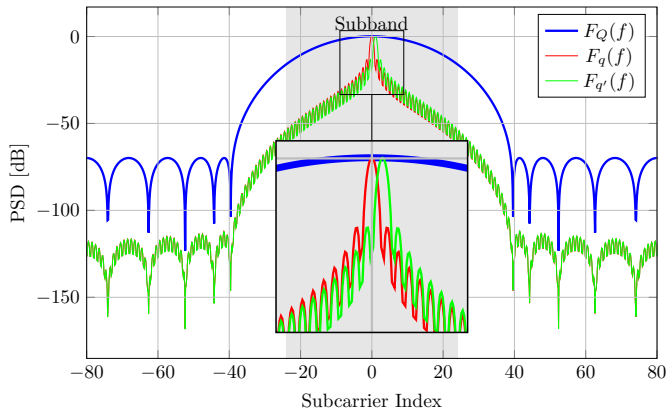


Fig. 2. Adjacent filters can be approximated by a simple shift in frequency. $N = 1024$, $Q = 48$, Dolph-Chebyshev-window with sidelobe attenuation of 70dB and length of $L = 74$ samples.

and DTFT denotes the discrete time Fourier transform. Assuming that $F(f)$ is designed far wider than a single subcarrier¹, the amplitude difference between two effective filters $F_q(f)$ and $F_{q'}(f)$ becomes smaller as $|q - q'|$ gets smaller, since only the farther sidelobes of $D_N(f)$ are affected by the filter $F(f)$. This behaviour is illustrated in Fig. 2. Assuming a linear-phase subband filter $F(f)^2$, the phase difference $\phi_{qq'}$ between $F_q(f)$ and $F_{q'}(f)$ is given by

$$\phi_{qq'} = \angle \left(\frac{F(q/N)}{F(q'/N)} \right). \quad (9)$$

Hence, the UF-OFDM signal can be approximated by dividing adjacent subcarriers into groups that use a common effective filter for modulation. Let \mathcal{Q} denote the number of subcarrier groups and let $\mathcal{Q}_i, i = 1, \dots, \mathcal{Q}$ be the set of subcarriers that belongs to the i th subcarrier group in each subband. Then, the UF-OFDM transmit signal can be approximated by

$$x[n] \approx \sum_{i=1}^{\mathcal{Q}} \sum_{k=0}^{K-1} \sum_{q \in \mathcal{Q}_i} s_{k,q} f_{q_i}[n] \exp \left(j2\pi \frac{K_0 + kQ + q}{N} n \right) \quad (10)$$

where $f_{q_i}[n]$ is the effective filter for the i th subcarrier group given by (6) and q_i is the subcarrier index that is used for the effective filter in the i th subcarrier group. The phase shift due to (9) is already to be compensated in $s_{k,q}$. However, the phase shift can also be corrected by the one-tap equalizer at the receiver side which is anyway necessary to compensate for the channel and subband filter phase rotation. This extra phase correction does not require additional complexity since it is known beforehand can hence included into the equalization process for the subband filter. Since $f_{q_i}[n]$ does neither depend

¹This assumption is valid, since the subband filter is supposed to jointly filter groups of subcarriers.

²Note that any FIR filter can be easily designed with linear phase.

on k or q it can be taken out of the sum as

$$x[n] \approx \sum_{i=1}^{\mathcal{Q}} f_{q_i}[n] \underbrace{\sum_{k=0}^{K-1} \sum_{q \in \mathcal{Q}_i} s_{k,q} \exp \left(j2\pi \frac{K_0 + kQ + q}{N} n \right)}_{s_i[n]}. \quad (11)$$

Now note that $s_i[n]$ is effectively the expression of an N -point IDFT, where inputs are only non-zero at the frequencies $\{K_0 + kQ + q\}_{q \in \mathcal{Q}_i}$. Observing that $n = 0, \dots, \tilde{N} - 1$ and exploiting the periodicity of the IDFT, the output of the IDFT simply needs to be periodically extended to compute $x[n]$ for $n \geq N$. The approximate system (11) can hence be understood as a windowed zero-padded OFDM system where different subcarriers are multiplied by a different window. A similar approach has been presented in [11] in the context of OOB reduction for CP-OFDM, showing a distinct relation between windowed-OFDM and UF-OFDM. However in the context of UF-OFDM the windowing is performed on a subband basis, creating several well-localized subbands, which benefits spectral agility. Note that (11) does not contain a costly convolution but merely a simple time-domain multiplication and hence offers the possibility for a reduced complexity implementation, as is assessed in the following section.

III. COMPLEXITY ANALYSIS

In this section we compare the complexity of the frequency-domain transmitter published in [10] with the present proposal. As a figure of merit we consider the number of required real-valued operations. Note that complex-valued addition and multiplications require 2 and 6 real operations, respectively. The number of real operations for an N -point (D)FFT is given in [10] as

$$\mathcal{F}(N) = \frac{34}{9} N \log N - \frac{124}{27} N - 2 \log N - \frac{2}{9} (-1)^{\text{ld } N} \log N + \frac{16}{27} (-1)^{\text{ld } N} + 8 \quad (12)$$

The number of real-valued operations for a baseline OFDM implementation is simply given by the complexity of the Fourier transform

$$\mathcal{C}_O = \mathcal{F}(N). \quad (13)$$

The overall complexity of the approximate frequency-domain signal generation proposal for UF-OFDM in [10] is given by

$$\mathcal{C}_F = \mathcal{F}(2N) + B[\mathcal{F}(N_0) + \mathcal{F}(2N_0) + 12N_0] + 4(B - 1)N_0, \quad (14)$$

where B is the number of allocated subbands and N_0 is parameter controlling the approximation accuracy that should be at least 64 for a subband width of $Q = 12$ [10].

As previously shown, the time-domain generation of the UF-OFDM signal in (11) can be implemented by performing a simple time-domain multiplication plus IDFT for each subcarrier group with subsequent addition of all groups. Accordingly, the number of required real operations is given by

$$\mathcal{C}_T = \mathcal{Q}(\mathcal{F}(N) + 6\tilde{N}) + 2(\mathcal{Q} - 1)\tilde{N}. \quad (15)$$

The complexity scales linearly with the number of subcarrier groups in each subband but is independent of the number of allocated subbands. The ratio between the present proposal and the OFDM baseline complexity approaches

$$\frac{C_T}{C_O} \approx Q \text{ for } N \rightarrow \infty. \quad (16)$$

IV. NUMERICAL ANALYSIS

In this section, the approximation accuracy of the proposed time-domain signal generation is numerically analyzed. Furthermore, exact operation counts for several UF-OFDM parameters settings are given and compared to both OFDM and the baseline algorithm in [10].

A. Approximation Accuracy

As the method in (11) only approximates the UF-OFDM signal compared to the exact signal in (1), it is mandatory to measure the deviation of the approximation from the exact signal. In particular, care should be taken in the analysis of the spectral properties of the transmit signal, as the low OOB emission of the UF-OFDM signal is a key ingredient for 5G applications. Let \mathbf{T}_e be the modulation matrix for the k_0 th subband given by

$$\mathbf{T}_e = \mathbf{F}_{k_0} \mathbf{V}_{k_0} \mathbf{N} \quad (17)$$

such that the exact transmit for the k_0 th subband is given by $\vec{x}_{k_0} = \mathbf{T}_e \vec{s}_{k_0}$. Further, let \mathbf{T}_F be the modulation matrix that modulates the data s_{k_0} according to the rule in (11), i.e. \mathbf{T}_F contains the \vec{f}_{q_i} , which are frequency-shifted to the correct subcarrier frequencies. Then, the mean-squared error (MSE) e between the exact and approximated signal is given by

$$e = \frac{1}{N} E[\|(\mathbf{T}_e - \mathbf{T}_F) \vec{s}_{k_0}\|^2] \quad (18)$$

$$= \frac{1}{N} E[(\mathbf{T}_e - \mathbf{T}_F) \vec{s}_{k_0})^H (\mathbf{T}_e - \mathbf{T}_F) \vec{s}_{k_0}] \quad (19)$$

$$= \frac{1}{N} \text{trace}\{(\mathbf{T}_e - \mathbf{T}_F)(\mathbf{T}_e - \mathbf{T}_F)^H\} \quad (20)$$

when transmitting unit-energy i.i.d. data symbols. Note that \mathbf{T}_e and \mathbf{T}_F both generate the normalized UF-OFDM signal. In order to produce non-normalized UF-OFDM signals, the inverse of the normalization matrix can be multiplied to the right of the modulation matrices. We have analyzed the MSE according to (18) for two different approximation configurations with details given in Tab. I. We have calculated the MSE for both normalized and non-normalized signalling. The according values are presented in Tab. II.

As shown, the approximation accuracy reduces as the subband size increases. This can be explained by the fact that with higher Q , more subcarriers are approximated by a single filter and hence the MSE increases. Additionally, the non-normalized signals are more accurate which is due to the fact that the subband filter attenuates the edge carriers more than the center ones. However, also the edge carriers are more prone to approximation error since they are farther away from the subgroup representative. Finally, the 3-Filter

TABLE I
PARAMETER SETTINGS FOR NUMERIC ANALYSIS

	Parameter	Symbol	Value
General	FFT Length	N	1024
	Subband Width	Q	{12, 24, 36, 48}
	Filter Length	L	74
	Filter Type	-	Chebyshev
	Filter Attenuation [dB]	-	{13, 32, 51, 70}
Single-Filter	Subcarrier groups	Q	1
	Subgroup Filter	q_i	$Q/2$
	Subgroups	Q_1	{0, 1, ..., Q - 1}
3-Filter	Subcarrier groups	Q	3
	Subgroup Filter	q_i	{ $\frac{Q}{6}, \frac{Q}{2}, \frac{5Q}{6}$ }
	Subgroups	Q_1	{0, ..., $\frac{Q}{3} - 1$ }
		Q_2	{ $\frac{Q}{3}, \dots, \frac{2Q}{3} - 1$ }
Q_3	{ $\frac{2Q}{3}, \dots, Q - 1$ }		

TABLE II
MSE e OF DEVIATION OF APPROXIMATED FROM EXACT SIGNAL FOR SINGLE FILTER AND 3-FILTER APPROXIMATION, USING NORMALIZED AND NON-NORMALIZED SUBBANDS.

Q	Single-Filter norm.	Single-Filter not norm.	3-Filter norm.	3-Filter not norm.
12	-41.47	-42.53	-49.76	-50.11
24	-36.65	-39.11	-44.47	-45.72
36	-33.23	-37.18	-40.25	-42.68
48	-30.35	-35.91	-36.48	-40.31

approximation achieves roughly 5dB improved accuracy compared to the single-filter approximation. However, all MSE are below -30dB, which means that the approximations deviate on average less than 0.1% from the exact signal. This value is clearly below e.g. the EVM requirement of 22dB for LTE [12].

Figs. 3 and 4 show the spectrum of a single subband for non-normalized and normalized subbands, respectively. Apparently, the subbands spectra get wider when the approximation is employed. This is clear, since the representative filters of each subcarrier group are taken from the center of the group. Accordingly, when they are shifted to the subband edge, the spectrum becomes wider. However, the change in the spectrum is negligible for the 3-filter expansion. On the other hand, since the single-filter expansion uses the center filter as the representative, it can achieve even lower sidelobes, however at the cost of a wider main lobe. This property even reduces the OOB emission in farther away frequencies.

B. Operation Count

The operation count for both the single-filter and 3-filter expansion and for the proposal in [10] are compared in Fig. 5, where the values are related to the OFDM complexity of equal N . As shown, the baseline approach from [10] requires a considerably increased amount of operations compared to OFDM, depending on the parameters of the signal. The single-filter and 3-filter approximations exhibit a relatively constant complexity, independent of the number of allocated

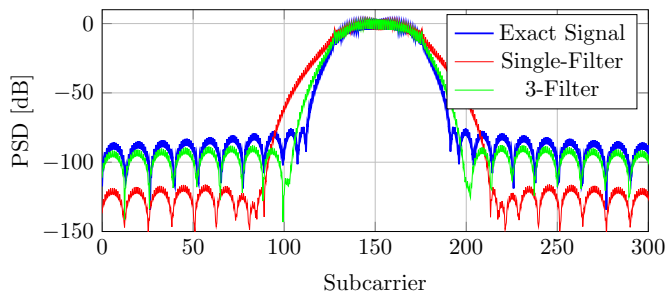


Fig. 3. Spectrum comparison for $Q=48$ and non-normalized subbands.

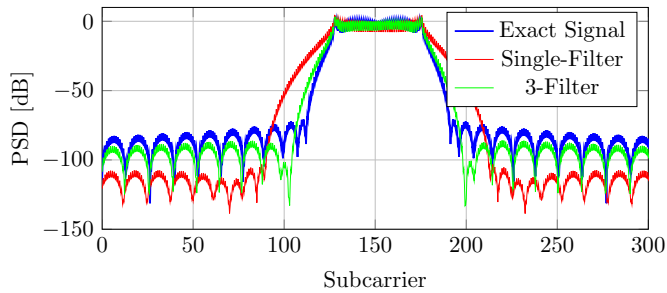


Fig. 4. Spectrum comparison for $Q=48$ and normalized subbands.

bands or subband bandwidth, which is far below the baseline complexity. Only, when a single subband is allocated, the baseline approach from [10] requires fewer operations than the 3-filter approach. The single-Filter approximation is only 20% more complex than an equivalent OFDM implementation which makes this approximation attractive for very low-cost devices that do not require too accurate signals or confined spectra.

V. CONCLUSION

This paper has provided a reduced complexity implementation of the UF-OFDM transmitter. The approximation exploits the similarity of adjacent effective subcarrier filters and can reduce the amount of required operations by replacing the filtering with a simple time-domain multiplication. Both the complexity and approximation accuracy were analyzed, using a single-filter and 3-filter UF-OFDM approximation. It was found that the MSE of the signal deviation is below -30dB for any parametrization and can be as low as -50dB for the 3-filter solution. The provided approximations can be implemented with only 1.2 and 3.6 times more operations than OFDM for the single-filter and 3-filter approximations, respectively, which is significantly below the complexity of previous works. The results can have significant impact on the costs and quality of upcoming UF-OFDM implementations.

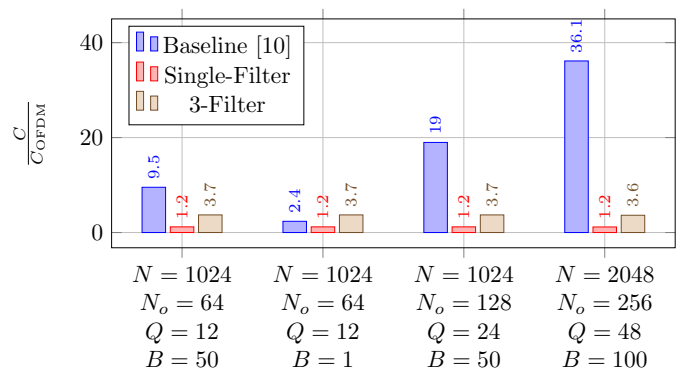


Fig. 5. Number of real operations for UF-OFDM transmitters related to equivalent OFDM transmitter.

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