

Complexity Reduction in Iterative MIMO Receivers Based on EXIT Chart Analysis

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Abstract

The application of the Turbo principle in MIMO receivers allows to achieve near-capacity performance with a number of different MIMO detection algorithms. However, this high performance is paid for by a substantial increase in complexity, as detector and decoder have to be run multiple times. In this paper, we show that an analysis of detector and decoder transfer curves in the EXIT chart can be used to select a suitable detection algorithm and parametrization of the outer channel decoder, such that the overall receiver complexity can be substantially decreased while retaining its original performance. In order to enable a flexible adaptation of the receiver parameters, we also propose a simple “measurement device” for mutual information that can be used to steer the parametrization, based on the quality of the soft information at the output of detector and decoder.

1 Introduction

Radio frequency spectrum is a scarce and thus valuable resource, which should be used as efficiently as possible. In order to attain this required high spectral efficiency, future wireless communications systems will make use of multiple antennae at transmitter and receiver to spatially multiplex several data streams into the same time-frequency bin. The main challenge for such multiple antenna (MIMO) systems lies in the non-orthogonality of the transmission channel, which makes correct separation of the data streams at the receiver a difficult task. This task can be solved in a very effective manner using Turbo processing, i.e., exchanging (extrinsic) information between inner MIMO detector and outer channel decoder. In this context, MMSE based “soft” successive interference cancellation [1], list sphere detection [2], and list sequential detection [3] are all known achieve performance close to the capacity limit of the MIMO channel while avoiding the prohibitive complexity of a full APP detector.

However, such detection strategies are still very complex and following this approach also requires running the detector and the outer channel decoder (which is usually one of the computationally most complex parts of the system) multiple times. On the other hand, it is well known that matching the transfer curves of the constituent “decoders” in the EXIT chart allows to increase the performance of an iterative system. It can therefore be expected that, if major gaps between detector and decoder transfer curves exist, these can be exploited to achieve similar receiver performance at substantially reduced complexity. We therefore study the transfer characteristics of a list sequential (LISS) [3] detector, a “Soft SIC” [1] detector and an outer “Turbo”

channel decoder, to find practical ways of achieving this aim. The remainder of this paper is organized as follows. In Section 2 we introduce the system model and describe the basic principles of iterative MIMO receiver architectures. Section 3 discusses the transfer characteristics of different MIMO detection techniques as well as the outer channel decoder and explains how this information can be used to reduce complexity. Section 4 introduces the new low-complexity measurement device for mutual information which can be used to flexibly adjust the parameters of the decoder and MIMO detector. In Section 5 we compare the performance and complexity of our proposed approach with that of conventional receiver strategies. We finally draw conclusions in Section 6.

2 System Model

2.1 MIMO-Model

We consider a MIMO system with M_{Tx} transmit and N_{Rx} receive antennae as depicted in Figure 1. Let $\mathbf{u}$ be a vector of information bits which are encoded by the outer encoder. The resulting bit stream $\mathbf{c}$ is interleaved and partitioned into blocks $\mathbf{x}_t$ containing $M_{Tx} \cdot L$ independent binary digits. Here, L is the number of bits per symbol, allowing to distinguish between $K = 2^L$ different constellation points. During the transmission process, for each time index t a single block $\mathbf{x}_t$ is mapped onto a $M_{Tx} \times 1$ complex vector of symbols $\mathbf{s}_t = (s_1, \dots, s_{M_{Tx}})^T$ whose components are chosen from some complex constellation $\mathcal{C}$ using the mapping function $s_t = \text{map}(\mathbf{x}_t)$ (for ease of nomenclature, this mapping function includes the assignment of bits to transmit antennae).

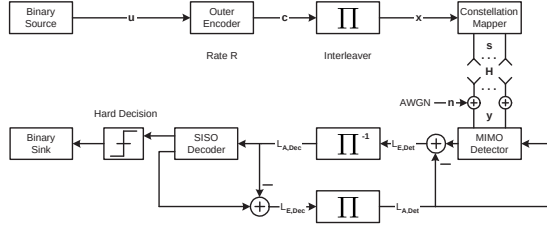


Fig. 1. Transmission model with encoder, MIMO channel and iterative receiver (soft-input soft-output MIMO detector and channel decoder).

The average total transmitted energy per (vector) symbol is normalized to $E[\|\mathbf{s}\|^2] = E_s$, and the energy is distributed evenly over the transmit antennae. Let $\mathbf{y}_t$ be the vector of received symbols at time index t , which is defined as

$$\mathbf{y}_t = \mathbf{H}_t \mathbf{s}_t + \mathbf{n}_t \quad (1)$$

where $\mathbf{H}_t$ is a dimension $N_{Rx} \times M_{Tx}$ MIMO channel matrix perfectly known at the receiver. Each entry of the channel matrix is an independent realization of a complex Gaussian random process with zero mean and variance $1/2$ per real dimension. The noise vector $\mathbf{n}_t$ consists of mutually independent zero-mean circularly symmetric complex Gaussian random variables, each with variance $N_0/2$ per real dimension.

2.2 Iterative MIMO Receivers

As receiver architecture we consider the serial concatenation of an inner MIMO detector and an outer channel decoder. Both detector and decoder are able to accept and generate soft information, which are exchanged between them during the iterative reception process. The detector uses the received signal, the channel state information and the a-priori information provided by the decoder to generate extrinsic information on the received bits. The channel decoder uses the correlation between different code bits introduced by the encoder (i.e., the code's structure) to generate extrinsic information about the information bits *and* the code bits. The latter information is interleaved and fed back to the detector which in turn employs the provided a-priori information to further improve its soft output.

For the representation of the soft information we use log-likelihood ratios (LLR). The LLR of a certain transmitted bit x_n conditioned on the received vector signal $\mathbf{y}$ (dropping time index t for ease of notation) is defined as:

$$L(x_n|\mathbf{y}) := \ln \frac{P[x_n = +1|\mathbf{y}]}{P[x_n = -1|\mathbf{y}]} \quad (2)$$

Using Bayes' theorem under the assumption of mutually independent bits x_n (which is justified by the use of an interleaver of appropriate size and structure), the joint probabilities can be split into products. The LLR

computation can now be rewritten as:

$$L(x_n|\mathbf{y}) = \ln \frac{\sum_{\mathbf{x} \in \mathbb{X}_{n,+1}} p(\mathbf{y}|\mathbf{x}) \cdot P[\mathbf{x}]/p(\mathbf{y})}{\sum_{\mathbf{x} \in \mathbb{X}_{n,-1}} p(\mathbf{y}|\mathbf{x}) \cdot P[\mathbf{x}]/p(\mathbf{y})} \quad (3)$$

where $\mathbb{X}_{n,\pm 1}$ is the set of $2^{M_{Tx} \cdot L - 1}$ bit blocks $\mathbf{x}$ with $x_n = \pm 1$, respectively. The MIMO channel introduces interference among the transmitted signals at the receiver. The conditioned probability density in (3) is thus given by the complex Gaussian distribution

$$p(\mathbf{y}|\mathbf{x}) = \frac{1}{(\pi N_0)^{R_x}} \exp \left(-\frac{1}{N_0} \|\mathbf{y} - \mathbf{H}\mathbf{s}\|^2 \right) \quad (4)$$

For the LLR computation only the exponential term is relevant – the constant scaling factor cancels out and can thus be omitted. The second term in (3) represents the a-priori knowledge fed to the detector from the outer decoder, whereas the third (bias) term $p(\mathbf{y})$ can be used to take the influence of different path lengths during the tree search into account. Note that this bias term effectively cancels itself out in the LLR computation, as only full length paths are considered there.

To evaluate the numerator and denominator in (3) it is often convenient to apply the so called “max-Log” approximation in order to speed up computation at the expense of some performance degradation [4]. Hence, the computation of the detector LLR can be approximated as a difference of two max-operations:

$$L(x_n|\mathbf{y}) \approx \max_{\mathbf{x} \in \mathbb{X}_{n,+1}} \left\{ -\frac{1}{N_0} \|\mathbf{y} - \mathbf{H}\mathbf{s}\|^2 + \sum_{n=1}^{M_{Tx}L} \ln P[x_n] \right\} - \max_{\mathbf{x} \in \mathbb{X}_{n,-1}} \left\{ -\frac{1}{N_0} \|\mathbf{y} - \mathbf{H}\mathbf{s}\|^2 + \sum_{n=1}^{M_{Tx}L} \ln P[x_n] \right\} \quad (5)$$

Evaluating the two max-operations in equation (5) by a brute-force approach (maximum likelihood detection, MLD) is well known to require an effort growing exponentially in the number of transmitted bits per vector symbol, i.e., the achieved (raw) spectral efficiency. However, there exist a number of suboptimal and close-to-optimal algorithms that achieve very good performance at only a fraction of the full MLD complexity. We will discuss some examples in the following section. For channel coding, we consider the parallel concatenation of two recursive systematic convolutional codes – a classical “Turbo code”. The decoder employs two instances of the BCJR [5] algorithm that exchange extrinsic information in a number of (internal) decoder iterations and afterwards feed back extrinsic information to the MIMO detector.

3 Transfer Characteristics of Detector and Decoder

Based on the recognition that only a few hypotheses $\mathbf{x} \in \mathbb{X}_{n,\pm 1}$ maximize each of the respective terms in (5), a number of suboptimal detection strategies

have been devised. Instead of a full enumeration, these algorithms construct a subset (list) $\mathbb{L} \subset \mathbb{X}$ to determine the LLRs. This subset should on the one hand include only a fraction of elements from $\mathbb{X}$ but on the other hand be large enough to allow to approach the true detector L-values as closely as possible. Two effective schemes are list sphere [2] (LSD) and list sequential [3] (LISS) detection. Successive interference cancellation [6] follows a different strategy. It considers only a single hypothesis, but takes the “interference” between different detected layers into account. The quality of the soft output of such a scheme can be substantially increased by taking error propagation effects into consideration, as proposed in [1]. As a prerequisite for all of these detection methods, we have to decompose the channel matrix $\mathbf{H}$ to obtain a matrix of upper triangular structure. Obviously, reliably received signals should be detected first, so we use the sorted QR decomposition [6] for this purpose.

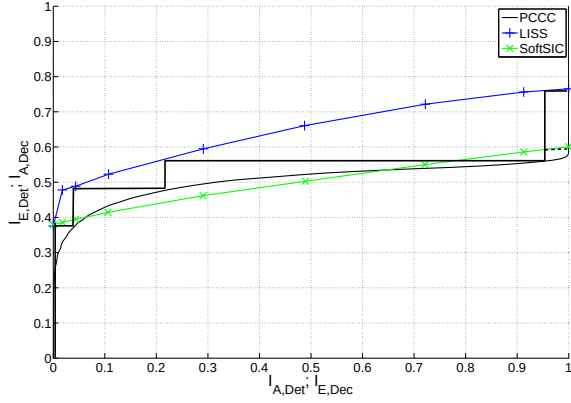


Fig. 2. Transfer characteristics of LISS and Soft SIC detector at $E_b/N_0 = 7\text{dB}$. Soft SIC and LISS provide the same extrinsic information for $L_A = 0$. A Soft SIC detector can be used instead of the LISS also for large values of L_A

Figure 2 shows the transfer characteristic for a LISS detector (stack size 512, 128 full length candidates, using path augmentation, no bias term) and a soft SIC detector. They show the typical behavior for Gray coded transmission, i.e., relatively slight increase in extrinsic information, with increasing a-priori knowledge. Two important facts can be observed: firstly, when no a-priori knowledge is available, the soft SIC detector provides the same amount of extrinsic information as the LISS detector. Secondly, as the transfer curve of the outer decoder is almost horizontal in the range around $I_{A,Dec} \approx 0.5$, there is a large gap between detector and decoder transfer curves when much a-priori knowledge is available at the input of the detector. This motivates for using a detector of much lower complexity, which will evidently produce lower quality soft output (i.e., a lower $I_{E,Det}$), but could still achieve the same performance as the LISS detector, if its transfer curve is sufficiently above that of the channel decoder for $L_{A,Det} \geq 0.5$. This idea is illustrated

by the example trajectory in Figure 2: the trajectory will start at the same point $I_{E,Det} = I_{A,Dec} \approx 0.37$ regardless whether we use a SoftSIC or LISS detector. We therefore can expect the overall trajectory (and receiver performance) to be unaffected by the selection of the detector strategy for the first iteration. It is also illustrated that in the fourth iteration, one may again use the SoftSIC detector, as the extrinsic information at its output is high enough to enable successful decoding (cf. the dashed horizontal line: $I_{E,Det} = I_{A,Dec} \approx 0.6$, thus $I_{E,Dec} \approx 1$ which is required to achieve very low error rates at the decoder output).

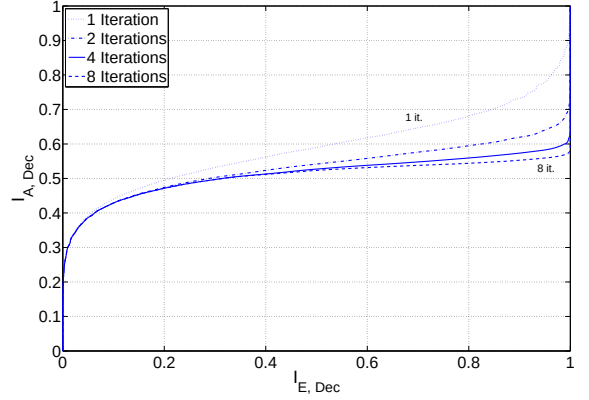


Fig. 3. Transfer characteristic of a PCCC based on $[7R, 5]$ constituent codes. The curve only depends on the information provided by the detector and is hence independent of SNR. It is clearly seen that the number of iterations has a major impact for $I_A \approx 0.5$.

For channel coding we focus our attention on the class of capacity-approaching codes. More specifically, we consider a parallel concatenated convolutional code (PCCC) with rate 1/2 memory 2 constituent encoders. For decoding we use the well known setup of a concatenation of two BCJR [5] decoders. Figure 3 shows the transfer characteristic of such a code, as a function of the number of internal decoder iterations. It can be seen that the transfer characteristic is largely independent of the number of iterations for a-priori knowledge below 0.4 and starts to (slowly) converge again as soon as the a-priori knowledge exceeds 0.6. As could be expected, the main decoding effort should hence be invested in the range around $I_{A,Dec} \approx 0.5$ – at values close to the capacity bound for rate 1/2 codes.

As can be seen from Figure 2, the receiver trajectory starts at values below $I_{A,Dec} = 0.4$ for SNRs close to the performance limit of the LISS detector (around 6dB). It therefore appears to be reasonable to scale the number of internal decoder iterations depending on the available a-priori information at the decoder input. This approach enables substantial complexity savings, especially when a high number of detector-decoder iterations is used. A detailed assessment of this technique, its combination with the flexible use of different detection algorithms will be presented in Section 5.

4 A Simple Measurement Device for Mutual Information

As explained in the previous section, measuring the mutual information I at the input/output of the channel decoder/MIMO detector allows to configure detector and decoder parameters such that they provide the desired quality of soft output at minimum effort. Several options exist to determine the value of I based only on the provided LLRs, without requiring any knowledge of the transmitted bits. It is typically assumed that the LLRs follow a Gaussian distribution, as depicted in Figure 4 and it can be shown that the mutual information can be “measured” as follows [7]:

$$I \approx 1 - \frac{1}{K} \sum_{k=1}^K H_b \left(\frac{1}{1 + e^{-|L_k|}} \right) \quad (6)$$

where $H_b(x) = -x \log_2 x - (1-x) \log_2 (1-x)$ is the binary entropy function. Evidently, using this approach involves the evaluation of non-linear functions for each LLR, which is computationally complex.

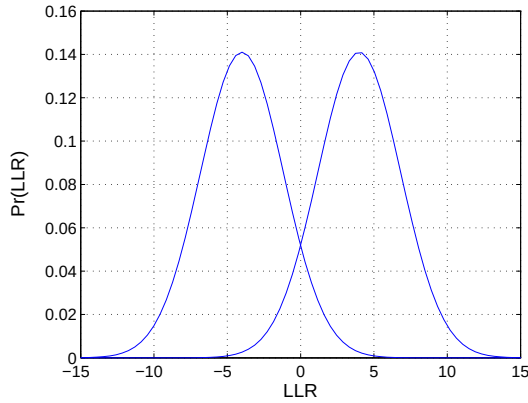


Fig. 4. Example distribution of LLRs with $L \sim \mathcal{N}(\pm\mu, \sigma^2)$, for $\mu = 4$ and $\Pr(x_n = 1) = \Pr(x_n = -1) = 1/2$, i.e., the coded bits are equiprobably distributed. Note that $\mu = \sigma^2/2$ and the distribution is hence fully described by the single parameter μ .

Let us therefore take a look again at the LLR distribution in Figure 4. Remembering the fact that for LLRs, the mean and variance of the distribution are related as $\mu = \sigma^2/2$, it is straightforward to see that the amount of mutual information contained in the LLRs can be inferred only from the single parameter μ .

A (quite simplistic) estimate $\hat{\mu}$ for μ can be established by using simple averaging over the absolute values of the LLRs:

$$\hat{\mu}_K := \frac{1}{K} \sum_{k=1}^K |L_k(\mu)| \quad (7)$$

Obviously, the overlap between the two distributions (for $x_n = 1$ and $x_n = -1$, cf. Figure 4) cannot be neglected for small values of μ and hence the approximation $\hat{\mu}_K \approx \mu$ is only valid for large values of μ . However, the value of the “biased estimate”

$\tilde{\mu}(\mu) := E\{|L(\mu)|\}$ can be derived analytically (please refer to the appendix for details):

$$\frac{\tilde{\mu}}{\mu} = 1 + \frac{2}{\sqrt{\mu\pi}} \exp(-\mu/4) - \operatorname{erfc} \left(\frac{\sqrt{\mu}}{2} \right) \quad (8)$$

with $\lim_{K \rightarrow \infty} \hat{\mu}_K = \tilde{\mu}$.

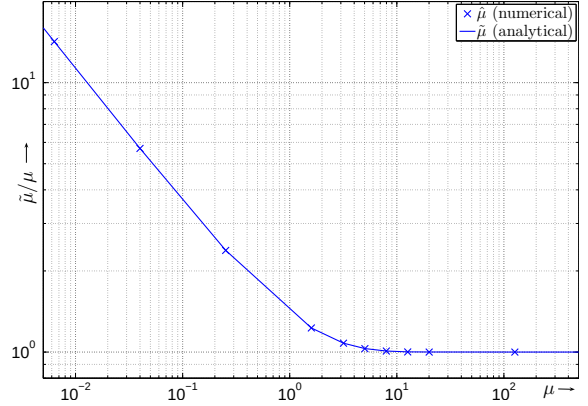


Fig. 5. Scaling function $\tilde{\mu}/\mu$.

Since the bias term is monotonously decreasing in μ , one can directly infer the value of μ from $\hat{\mu}$, map this, for example, to the “SNR” on the LLRs and then on the contained mutual information via the standard capacity formula of the binary AWGN channel. In practice, a direct mapping from $\tilde{\mu}$ to I via an appropriate look-up-table would be the most convenient solution to determine I from a “measured” $\hat{\mu}$.

5 Simulation Results

The performance of different approaches for reducing the complexity of iterative MIMO receivers was evaluated by simulating a 4×4 MIMO system using 16-QAM transmission.

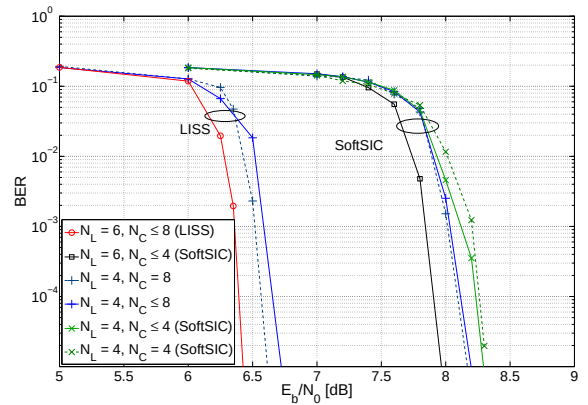


Fig. 6. Performance results of a LISS and a soft SIC receiver, when using fixed (dashed curves) and variable number of internal decoder iterations (solid curves). Performance is essentially unaffected by introducing this complexity reduction technique.

We assumed the receiver to know the channel perfectly and the channel gains to remain constant over

the transmission of one vector symbol but change statistically independent from one symbol to another. As channel code we used a rate $R = 1/2$ PCCC with generator polynomials $G = [7_R, 5]$ for the constituent codes. The information block size was 9214 bits (9216 including tail bits), allowing direct comparison with the results from [3], [2].

Figure 6 shows the performance using “conventional” LISS and Soft-SIC detectors, with fixed and variable number of internal decoder iterations N_C . N_L denotes the number of detector-decoder iterations. As is evident, the performance of such a setup remains largely unaffected by using a variable number of N_C . The SNR loss by using a SoftSIC instead of a LISS detector is around 1.5 dB.

To assess the expected reduction in decoding complexity, Figure 7 shows the average total number of internal decoder iterations. If we consider 8 iterations as the standard case, a decoding complexity reduction of around 50% can be achieved over the whole range of SNRs. If some loss in performance is acceptable (cf. Figure 6) and only 4 iterations are used, a complexity reduction is only possible outside the waterfall region. This is explained by the fact that in the waterfall region, the main part of the detection-decoding trajectory between SoftSIC and PCCC will be in the region around $I_{A,Dec} \approx 0.5$ (cf. Figure 2), where further reductions in the iteration number result in significant residual errors.

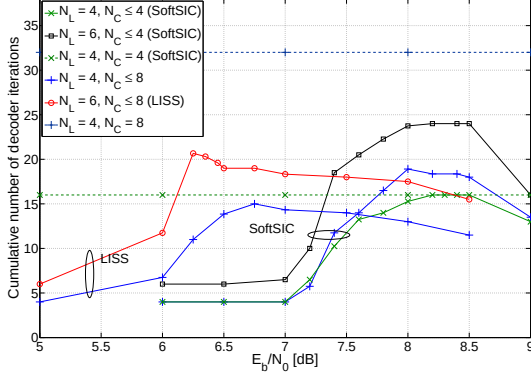


Fig. 7. Average sum of internal decoder iterations when using a LISS and a soft SIC receiver. Decoding complexity can be reduced by more than 50% (compared to the case when a fixed number of 8 internal decoder iterations is used).

However, the true potential of the proposed approach is visible when aiming for high performance, e.g., by using a LISS detector and a higher number of detector-decoder iterations. Letting $N_L = 6$ pushes the waterfall region to around 6.3 dB while the total number of decoder iterations rises to only around 20 (compared with 48 in the full complexity setup). Note that the *maximum* and the *average* number of decoder iterations are very similar at a specific SNR value, which is important for practical implementation. This is due to the fact that the proposed complexity reduction exploits the properties of the PCCC *transfer characteristic* (which depend

only on the SNR), instead of using “stopping criteria” (which depend on the currently received data/channel conditions).

However, we would also like to decrease *detection* complexity. As stated in Section 3, making flexible use of a LISS and a SoftSIC detector appears to be a promising option. We investigated two different setups: in the first version ($N_{SIC} = 2$) we always use the SoftSIC in the first and the last of four detector-decoder iterations. In the second setup ($N_{SIC} \geq 1$), we always use the SoftSIC in the first iteration, but only use it in the last iteration whenever $I_{A,Det} \geq 0.65$ (as measured by our tool). The performance of the two setups is depicted in Figure 8.

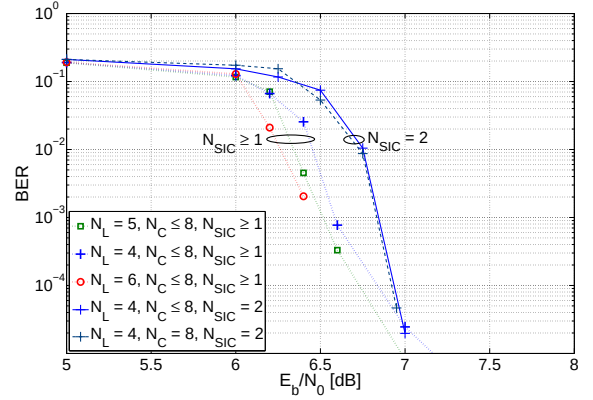


Fig. 8. Performance of a combined LISS-SoftSIC receiver. Using the SoftSIC at the end of the iterative process results in error floor effects in the region between 6.5 and 7 dB.

Again, using a fixed (dashed curve) and variable (solid curve) number of internal decoder iterations yields the same performance, while the latter enables to substantially reduce complexity. The average number of iterations does not exceed 16 for the 4 decoder-iterations case and 20 for the 6 iterations case (results not shown), similar to the case with fixed detection strategy (cf. Figure 7).

However, using the SoftSIC in the last iteration evidently results in a performance decrease – around 0.3dB SNR loss for the $N_{SIC} = 2$ setup, and error floors in the region between 6.5 and 7 dB for the $N_{SIC} \geq 1$ setup. The SoftSIC is evidently a too optimistic choice at such low SNRs, as is confirmed by the EXIT chart in Figure 2. The transfer curve of the SoftSIC detector is only slightly above the PCCC transfer curve at 7dB – for all values below that point, using the SoftSIC receiver will yield insufficient soft information to achieve error-free decoding with the PCCC decoder. Evidently, using a LISS detector with lower number of full length candidates (e.g. only 32) in the last iteration would be a better solution for SNRs below 7dB. We are currently investigating this option.

Note, however, that the $N_{SIC} = 2$ setup decreases detection complexity by almost 50% (the SoftSIC complexity is negligible w.r.t. the LISS complexity [8]) and

that a LISS of similarly low complexity would use 32 full length candidates and its waterfall region is only at 7.3 dB [8]. As outlined before, the obtained complexity reduction can also be invested into a higher number of detector-decoder iterations and thus a higher achievable performance at original receiver complexity. In this context, using 5 detector-decoder iterations and a SoftSIC only in the first iteration appears to be a promising approach towards increasing performance while retaining receiver complexity.

6 Conclusions

In this paper, we considered the use of EXIT chart analysis for complexity reduction in iterative MIMO receivers. An analysis of LISS and SoftSIC detector transfer curves and their relation to the transfer curve of a PCCC decoder showed that in the first detector-decoder iteration it is possible to use a SoftSIC instead of a LISS detector, without experiencing any losses in performance. A SoftSIC may also be used in the last iteration, for SNRs beyond 7dB. For lower SNRs, a LISS with a low number of full-length candidates (e.g. 32) appears to be a better choice. We are currently studying this option.

It is also possible to substantially reduce the total number of PCCC decoder iterations by measuring the a-priori knowledge at the output of the detector and exploiting the fact that the amount of mutual information generated by the decoder is essentially independent of the number of internal iterations for $I_{A,Dec} < 0.4$. This is especially true when a high number of detector-decoder iterations is used in a LISS setup. In this context, we proposed a low-complexity measurement tool that can be used to measure the mutual information at the output of detector and decoder.

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APPENDIX

We assume the LLRs as the output of detector and decoder to be Gaussian distributed with mean and variance related as $\mu = \sigma^2/2$ [7]. In this case, the distribution of the LLRs is defined by:

$$p_L(l|X=x) = \frac{1}{2\sqrt{\pi\mu}} \exp(-(l-x\mu)^2/4\mu). \quad (9)$$

As explained in Section 4 we use the quantity $\tilde{\mu} = E\{|l|\}$ to determine I . Since the two LLR distributions are symmetrical for $x = +1$ and $x = -1$,

the mean can be calculated by considering only one of the cases (e.g., $x = +1$) as follows (cf. Figure 4):

$$\begin{aligned} \tilde{\mu} &= \int_{-\infty}^0 -l p_L(-l)dl + \int_0^{\infty} l p_L(l)dl \\ &= \int_0^{\infty} \frac{l}{2\sqrt{\pi\mu}} \exp\left(-\frac{(l-\mu)^2}{4\mu}\right) dl \\ &\quad + \int_0^{\infty} \frac{l}{2\sqrt{\pi\mu}} \exp\left(-\frac{(l+\mu)^2}{4\mu}\right) dl. \end{aligned} \quad (10)$$

The consistency property $p_L(-l) = \exp(-l)p_L(l)$ may be equivalently used to derive this formulation. For shortness of space, we treat the two cases $l + \mu$ and $l - \mu$ jointly. Defining the shorthand notation $f(l) = \exp(-(l \pm \mu)^2/4\mu)$, our task is now equivalent to solving

$$\begin{aligned} E(\pm\mu) &= \frac{1}{2\sqrt{\pi\mu}} \int_0^{\infty} l f(l) dl \\ &= \frac{-2\mu}{2\sqrt{\pi\mu}} \int_0^{\infty} -\frac{2(l \pm \mu)}{4\mu} f(l) \mp \frac{1}{2} f(l) dl \\ &= -\sqrt{\frac{\mu}{\pi}} \left([f(l)]_0^{\infty} \mp \sqrt{\mu\pi} Q\left(\pm\sqrt{\frac{\mu}{2}}\right) \right) \\ &= \sqrt{\frac{\mu}{\pi}} \exp\left(-\frac{\mu}{4}\right) \pm \mu Q\left(\pm\sqrt{\frac{\mu}{2}}\right), \end{aligned} \quad (11)$$

where we used the substitution $u = (l \pm \mu)/\sqrt{2\mu}$ to solve the second part of the integral, $\int f(l)dl$. From (11) we now have

$$\begin{aligned} \tilde{\mu} &= \mu \left(Q\left(-\sqrt{\frac{\mu}{2}}\right) - Q\left(\sqrt{\frac{\mu}{2}}\right) \right) + 2\sqrt{\frac{\mu}{\pi}} e^{-\frac{\mu}{4}} \\ &= \mu \left(1 - \operatorname{erfc}\left(\frac{\sqrt{\mu}}{2}\right) + \frac{2}{\sqrt{\mu\pi}} \exp(-\mu/4) \right). \end{aligned}$$

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