Compressed Sensing for Feedback Generation in OFDM Based LiFi Systems

Javad Gholipour*, Kai Lennert Bober**, Malte Hinrichs**, Volker Jungnickel**
*Vodafone Chair Mobile Communications Systems, Technische Universitat Dresden (TUD), Germany
**Metro, Access and In-house Systems Group, Photonic Networks and Systems Department,
Fraunhofer Heinrich Hertz Institute (HHI), Germany
gj_javad@yahoo.com, kai.lennert.bober@hhi.fraunhofer.de, malte.hinrichs@hhi.fraunhofer.de,
volker.jungnickel@hhi.fraunhofer.de

***Abstract*—In this paper, we evaluate the effectiveness of compressed-sensing-based channel estimation in LiFi orthogonal frequency-division multiplex (OFDM) systems with multiple-input multiple-output (MIMO). The sparseness of LiFi channels suggests that it is beneficial to utilize compressed sensing which has been investigated also for radio communications. In this paper, we formulate the mathematical problem of compressed-sensing for feedback generation in OFDM-based LiFi systems. We use well-known algorithms for both, sparse pilot design and sparse recovery and apply them to LiFi OFDM channels. We evaluate the effectiveness of these algorithms by measuring the achievable mean squared error (MSE) and vary the total numbers of subcarriers as well as the number of pilot subcarriers.**

***Index Terms*—LiFi, compressed sensing, channel estimation, OFDM, MIMO**

I. Introduction

LiFi is particularly suitable for Internet of things (IoT) applications in industrial and medical scenarios, because it operates in the optical domain, which is robust against electromagnetic interference. Unlike radio waves, which usually experience rich scattering, optical propagation mostly uses the line-of-sight (LoS). This simplifies some aspects of the system design (channel estimation, feedback design, precoding). On the other hand, other aspects such as potential blockage of the LoS become more relevant.

To design a practical system, algorithms need to support mobility, must therefore be processed in realtime, as blockages can occur rather quickly, i.e. in few milliseconds. One of the most challenging tasks is to design an effective feedback scheme which has high performance and is technically feasible at the same time.

LiFi systems use intensity modulation and direct detection (IM/DD). In this way, the information is encoded in the intensity of light and then retrieved through a photodetector (PD) at the receiver. Nowadays, light-emitting diodes (LEDs) are the commonly used emitters. In IM/DD, the modulated light intensity must be non-negative and real-valued. With good LED drivers, bandwidth can be up to few hundred MHz.

This research was partially funded by the German Ministry of Research and Education (BMBF) in the project SESAM under grant agreement No. 16KISO639K and European Union Horizon 2020 research and innovation program under grant agreement No. 825651 (ELIoT).

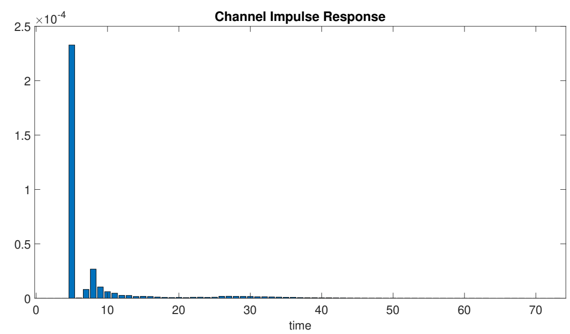

Fig. 1. Channel impulse response of a typical LiFi system from the standardization project IEEE 802.15.7. The channel length is L=73.

To reach Gbit/s, high spectral efficiency is needed which can be realized by using OFDM in LiFi systems. OFDM allows each subcarrier to contribute optimally by adaptive bit and power loading.

As mentioned before, in IM/DD, the modulated light intensity must be non-negative and real-valued. The common method to obtain a real-valued signal, is to enforce Hermitian symmetry on the subcarriers after inverse fast Fourier transform (IFFT). The non-negativity condition can be satisfied by several techniques. One of those methods is DC-biased optical OFDM (DCO-OFDM) [1], in which a positive direct current (DC) bias is added. Although this technique leads to increased power consumption, it allows high spectral efficiency.

In communication systems, pilot symbols are usually embedded into the data for channel estimation. In pilot-based channel estimation for multiple-input multiple-output (MIMO), orthogonal frequency or time domain training is used to estimate each single single-input single-output (SISO) channel [2] in parallel with others. In these methods, selected subcarriers which are known by both transmitters and receivers, are used for channel estimation. The pilot design should be done carefully as it depends on the dynamics of the channel. While pilot distances are commonly designed based on Nyquist criteria, in case the channel is sparse, other techniques can be used which results in less overhead.

The main assumption in the compressed sensing based channel estimation is that the channel can be represented in some basis as compressible. In traditional Nyquist sampling, the sampling rate must be at least twice as the highest frequency component of the signal. But in compressed sensing, fewer samples are needed, as they are proportional to the information contained in the signal [3], [4]. The fact that the LiFi channels mainly rely on the LoS (Figure 1), provide the possibility to adopt compressed sensing techniques for channel estimation.

The pilot design plays a crucial role for compressed sensing based channel estimation. The reason is that the measurement matrix which depends on both, the pilot pattern and the pilot amplitudes, dominates the estimation performance. It was proven that random pilot patterns can be the perfect solution for sparse channel estimation [5]–[7], but it is not practical for realistic channel estimation [8], [9].

For SISO orthogonal frequency-division multiplex (OFDM) systems, deterministic pilot designs have been studied intensively [9]–[17] by using a uniform pilot power allocation. The main focus is to find search strategies to obtain pilot patterns based on some criteria, such as mutual coherence and its modified version [10], [18], [19]. Among the search strategies, the discrete stochastic approximation [11], [12] and a genetic algorithm [16] attracted significant attention.

In [14], [20]–[22], pilot design for the multiple-input-single-output (MISO) OFDM channel is studied based on mutual coherence. The idea is to simplify the problem by using orthogonal pilot patterns for different transmitters. Therefore, the MIMO channel estimation problem is simplified to several SISO channel estimation sub-problems. The second benefit of this approach is to avoid mutual interference for different transmitters during the channel estimation process.

Sparse recovery algorithms reconstruct a sparse signal from undersampled measurements. These algorithms are classified as convex relaxations, non-convex optimization and greedy algorithms. Convex relaxations algorithms solve a convex optimization problem. They use techniques such as projected gradient, interior-point or iterative thresholding [23]. Non-convex optimization techniques assume that the knowledge about the signal distribution is known in advance. Based on this knowledge, the signal can be recovered [24]. But the computational complexity increases intensively [25]. An example of the greedy algorithms is Orthogonal Matching Pursuit (OMP). This algorithm tries to find the global optimum by selecting a local optimum in each iteration. In [26], [27], several variants of OMP are proposed.

Although the compressed sensing methods have been used a lot in radio wave communication, their use is new in optical wireless communication. In [28], [29], those methods were utilized to estimate the channel in visible light communication systems.

The main objective of this paper is a feedback scheme for LiFi systems based on the above mentioned features. One of the most critical parameters, related to the quality of such a scheme, is the overhead. The overhead can be minimized by optimized channel estimation and feedback provisioning. Our focus in this paper is on compressed sensing based channel estimation which has two parts. First, we consider the pilot-based channel estimation for a multi-user MIMO-OFDM LiFi system. Aiming at high performance, we allocate the pilots for each transmitter at disjoint sets of subcarriers, yielding a sparse pilot design. Thereby, the MIMO channel estimation problem is simplified to multiple SISO channel estimation problems. Next, we focus on decreasing the feedback overhead by exploiting the intrinsic sparsity of LiFi channels, due to the dominant LOS propagation. In mathematical language, the channel is sparse or at least compressible in the Fourier basis. This offers the opportunity to use compressed sensing to estimate the channel with fewer measurements and lower errors compared to conventional estimators used for radio systems, where propagation is dominated by non-LOS.

II. Problem Formulation

A. LiFi MIMO OFDM Channel Model

The following MIMO OFDM channel definition comes from [30]. Consider a MIMO-OFDM system, in which the transmitter and the receiver are equipped with n_T LEDs and n_R PDs, respectively. Each LED uses N subcarriers to send its information. For each LED, N_P subcarriers are used as pilot to estimate the channel at the receiver. The assigned pilot subcarriers to the i-th LED are denoted as $P_i = \{P_{i,1}, P_{i,2}, ..., P_{i,N_P}\}$, with the assumption that $1 \leq P_{i,1} < P_{i,2} < ... < P_{i,N_P} \leq N$. The LEDs use orthogonal pilots to be able to have interference free channel estimation. The orthogonality of pilots in the frequency domain means that $P_i \cap P_j = \emptyset$ for $1 \leq i \neq j \leq n_T$. This assumption leads to changing the MIMO-OFDM channel estimation into estimation of $n_T \times n_R$ SISO-OFDM channels. The pilot symbols transmitted by i-th LED are represented as $x_i = [x_i(1), x_i(2), ..., x_i(N_P)]^T$, which is received at j-th PD as $y_{j,i} = [y_j(p_{i,1}), ..., y_j(p_{i,N_P})]^T$. The channel input-output relation between the i-th LED and the j-th PD is:

$$y_{j,i} = X_i F_i h_{j,i} + n_{j,i}. \tag{1}$$

Here, $h_{j,i}$ is the channel impulse response between the i-th LED and the j-th PD which is represented as $h_{j,i} = [h_{j,i}(1), ..., h_{j,i}(L)]^T$. $X_i = diag\{x_i(1), x_i(2), ..., x_i(N_P)\}$ is a diagonal matrix which its diagonal elements show the pilot powers at the pilot subcarriers assigned to the i-th LED. $n_{j,i} = [n_j(p_{i,1}), n_j(p_{i,2}), ..., n_j(p_{i,N_P})]^T$ shows the additive white Gaussian noise (AWGN) at the pilot subcarriers assigned to the i-th LED, which is modeled as $n_{j,i}\ CN(0, \sigma^2 I_{N_P})$. F_i is a discrete Fourier transform (**DFT**) submatrix whose elements are defined as $[F_i]_{n,l} = e^{-j2\pi N(n-1)(l-1)}$ for $n \in P_i$ and $l \in \{1, ..., L\}$. It is needed to say that here the **DFT** submatrix is a modification of the **DFT** matrix by selecting only the rows corresponding to the pilot subcarriers of the i-th LED. Defining $\Phi_i = X_i F_i$ as the measurement matrix for the i-th LED, the channel input-output relation (1) can be rewritten as:

$$y_{j,i} = \Phi_i h_{j,i} + n_{j,i}. \tag{2}$$

The LiFi channel impulse response typically has few taps (Figure 1). Therefore, we assume that the channel $h_{j,i}$ is a k-sparse vector of length L, meaning that $h_{j,i}$ has at most k

non-zero elements where $k \ll L$. Therefore, the compressed sensing theory can be applied for estimation of $h_{j,i}$ with significantly less number of pilots than conventional methods.

B. LiFi MIMO OFDM Sparse Channel Estimation

Recall that in a MIMO OFDM LiFi system, using orthogonal pilots for the channel estimation, the channel input-output relation between the i-th transmitter and the j-th receiver can be written as:

$$y = \Phi h + n. \tag{3}$$

To avoid confusion, we ignored i and j indices. The goal is to find the sparsest h such that its mean squared error gets sufficiently small. Mathematically it can be formulated as:

$$min\|\hat{h}\|_0 \quad s.t. \quad \|y - \Phi\hat{h}\|_2^2 < \epsilon \tag{4}$$

To allow h to be reconstructed uniquely from y it is required that the measurement matrix satisfies the restricted isometry property (RIP) [31]. The main issue with this condition is that there is no polynomial time algorithm to check whether a matrix satisfies RIP.

Definition 1: Matrix Φ has RIP, if:

$$(1-\delta)\|h\|_2^2 \leq \|\Phi h\|_2^2 \leq (1+\delta)\|h\|_2^2 \quad and \quad \delta \in (0,1) \tag{5}$$

Although checking the satisfaction of RIP for a matrix is NP-hard, there exists an alternative condition to guarantee the perfect reconstruction. This condition is based on the coherence of the measurement matrix which is defined as the maximum cross-correlations between its normalized columns. Checking this condition is a feasible problem and therefore, it is used in many sparse channel estimation problems.

Definition 2: For a given pilot pattern

$$p = \{p_1, p_2, \ldots, p_{N_p}\} \tag{6}$$

where $1 \leq p_1 < p_2 < \ldots < p_{N_P} \leq N$, the coherence of the measurement matrix Φ is defined as the maximum absolute cross correlation between any pair of its different columns:

$$\begin{aligned} g(p) = \max_{0\leq m<n\leq L-1} |\langle \Phi(m), \Phi(n) \rangle| \ = \\ \max_{0\leq m<n\leq L-1} \left| \sum_{j=1}^{N_p} |x(p_j)|^2 \, \omega^{p_j(n-m)} \right| \end{aligned} \tag{7}$$

Here, $\Phi(m)$ shows the m-th column of Φ and $\omega = e^{-j2\pi/N}$.

Theorem 1: The sparse vector h can be reconstructed perfectly, if the coherence of the measurement matrix μ_Φ, is less than $1/2k$, where k is the sparsity of h [32].

Therefore, the pilot design problem can be explained mathematically as the measurement matrix coherence minimization problem. Formally, it can be written as:

$$p_{opt} = \arg\min_p g(p) \tag{8}$$

Let's assume that all OFDM pilot subcarriers have equal powers:

$$|x(p_1)|^2 = |x(p_2)|^2 = \cdots = \left|x\left(p_{N_p}\right)\right|^2 = E. \tag{9}$$

For simplicity, let $c = n - m$. Therefore:

$$g(p) = E \cdot \max_{1\leq c\leq L-1} \left| \sum_{i=1}^{N_p} \omega^{p_i c} \right|. \tag{10}$$

III. Simulation Framework

We use MATLAB to simulate the pilot designing and channel estimating algorithms.

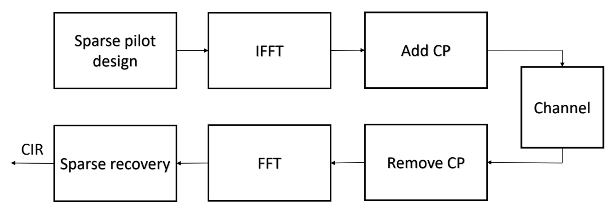

Fig. 2. Block diagram of the estimating system

In this paper we want to evaluate compressed sensing based channel estimation methods in LiFi OFDM system. As it is shown in Figure 2, the simulations start with sparse pilot design for which we use the stochastic sequential search (SSS) for SISO channels and modified version of SSS for MISO channels from [14]. After that, we do IFFT and then add a cyclic prefix to the signal. The lengh of the cyclic prefix is a quarter of the OFDM symbol lenght. The next step is to send the signal into the noisy LiFi channel. At the receiver side, we do the reverse procedure. First we remove the cyclic prefix and then do fast Fourier transform (FFT). In the last step, the OMP sparse recovery algorithm is applied to the signal.

To evaluate the performance of the algorithms we plot the MSE changes by SNR. During our evaluation, we show the effect of total number of subcarriers and total number of pilot subcarriers on MSE.

Our channel model is based on the channel simulator provided at HHI for LiFi system. We also do simulation for other random sparse channels including 1-sparse 2-sparse and other sparse channels.

In our simulation the total number of subcarriers is 128. We also do comparison with the cases where the total number of subcarriers are 256, 512, and 1024 to be able to see the effect of the total number of subcarriers on the performance of the algorithms.

As the baseline, we use linear estimator.

In all simulations, we assume the uniform pilot power allocation, then the goal of the pilot designer is to find the

Pilot design	SSS, SSS for MISO, CDS, Equally-spaced pilots
Recovery algorithm	OMP, Linear estimator
N	128, 256, 512, 1024
N_P	8, 16, 32, 64, 128
L	73
Sparsity	1, 2, 7, LiFi compressible channel
Cyclic prefix	$1/4N$
SNR range	-5 dB - 25 dB

TABLE I
SIMULATION PARAMETERS

best possible pilot pattern based on the design criterion. The simulation parameters can be seen in the table I.

We use the mentioned simulation framework to obtain the results of the next section.

IV. RESULTS AND DISCUSSION

In this section we do our simulation based on SSS for pilot design and OMP as the recovery algorithm. As a baseline, we use the linear estimator as well as CDSs. In simulations, we consider different sparse channels.

We start our analysis by evaluating the effect of the number of pilot subcarriers N_P on the MSE for a fixed number of subcarriers N. Then we compare the results of SSS algorithm and CDS to see efficiency of SSS. After that, we evaluate the effect of N on MSE for a fixed number of N_P. In the next step, we analyze the effect of different initialization of pilot sequence generator of SSS (different random start sequences), on MSE. We do also such evaluation for a typical compressible LiFi channel as well as some sparse random channels and compare the effectiveness of SSS and OMP with different equally-spaced pilot patterns and linear estimator. The last step is to apply such methods for MISO channel and compare the results and evaluate the fairness of the algorithms for different LEDs.

A. The Effect of the Number of Pilot Subcarriers on the Performance of SSS

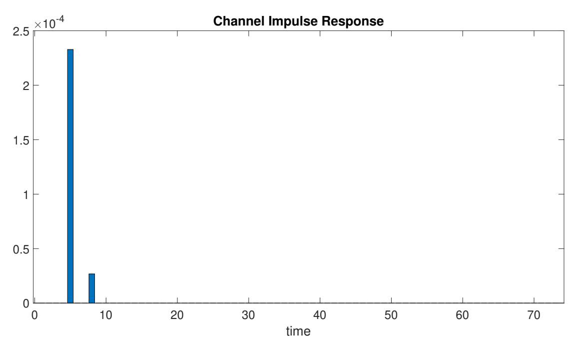

Fig. 3. Channel impulse response of a 2-sparse channel with L=73

Consider a 2-sparse channel as depicted in Figure 3. For this channel, we want to compare the effect of selecting different N_P, on the performance of SSS for a fixed N which is 128. We use OMP algorithm [18] for the channel estimation part.

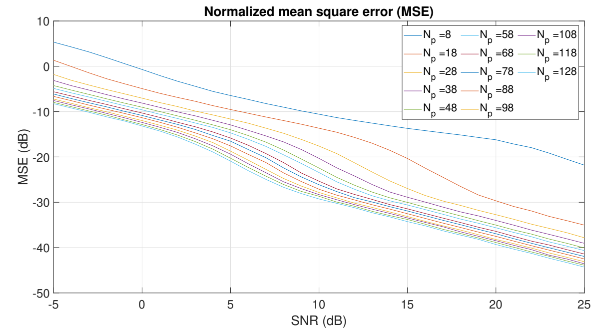

Fig. 4. The effect of N_P on the MSE for SSS. N is 128 and $N_P = [8 : 10 : 128]$

As we can see in Figure 4, by increasing N_P, the MSE decreases at each SNR. The reason is that using more pilot subcarriers, provides more information about the channel and therefore the estimation error decreases.

B. The Effect of the Number of Subcarriers on the Performance of SSS

Consider again the 2-sparse channel as depicted in Figure 3. For this channel, we want to compare the effect of selecting different N, on the performance of SSS for a fixed N_P which is 15. We use OMP algorithm [18] for the channel estimation part.

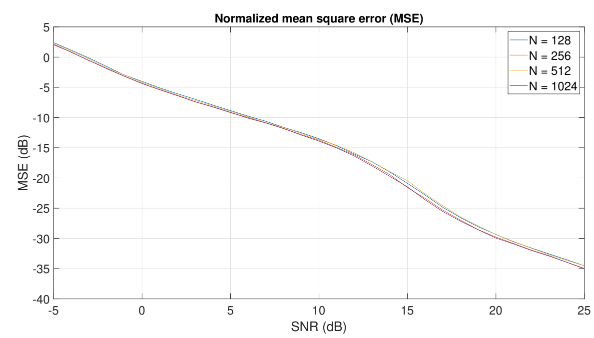

Fig. 5. The effect of N on MSE for SSS. N_P is 15 and $N = 128$, 256, 512, and 1024

It is obvious from the Figure 5, that different number of subcarriers do not affect the MSE significantly.

C. The Effect of Different Stopping Criteria on the Performance of OMP

OMP can have two different stopping criteria, e.g. sparsity of its output or the residual level. In our previous simulations, we used the sparsity as the stopping criterion of OMP. For the 1-sparse channel, we used sparsity 1 and for the 2-sparse channel, we used sparsity 2 as the stopping criterion of OMP. In this subsection, we evaluate the effect of choosing different stopping criteria on the performance of OMP.

In our simulations, we use a more realistic channel impulse response which is provided by the LiFi channel simulator of the Photonic Networks and systems department of Fraunhofer HHI (Figure 1).

In our first simulation, we compare the effect of selecting different sparsities as the stopping criterion for OMP. We also compare them with the performance of a linear estimator which uses equally-spaced pilots.

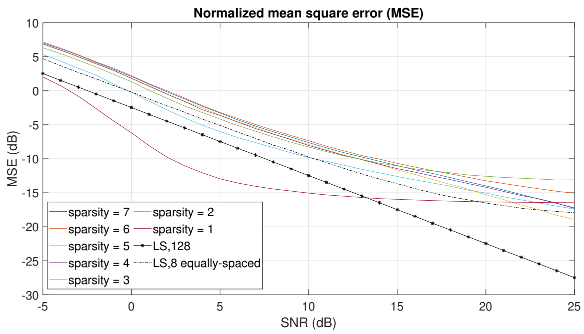

Fig. 6. The effect of the different sparsities as the stopping criterion for OMP, on the MSE for SSS. N is 128 and N_P is 8. The comparision is also done with the MSEs of the LS estimators with 8 equally-spaced pilots as well as the whole pilot subcarriers.

In Figure 6, we can see at low SNRs, choosing sparsity 1 as the stopping criterion, outperforms higher sparsities. The reason is that, at low SNRs, minor taps of the CIR are at the noise level, but at high SNRs, those minor taps are considerable in comparison with noise. Therefore, at high SNRs, the minor taps cannot be ignored and must be estimated by increasing the sparsity level of OMP, otherwise we have error.

If we compare the results with the output of the linear estimator with 8 equally-spaced subcarriers, it is obvious that OMP with sparsity 1, almost outperforms it. If we also compare it with the linear estimator which uses all subcarriers as pilots, we can see that OMP with sparsity 1, outperforms it at low SNR until 12.5dB.

In our second simulation, we use the more realistic stopping criterion, that is, residual level. The reason to use this stopping criterion, is that, we usually don't have exact knowledge about the channel sparsity at the receiver side. We also make a comparison with the results of the linear estimator which uses equally-spaced pilots.

In Figure 7, we can see the MSEs for different N_P. If we compare the results of this figure with the the Figures 6, we can see the modification of the MSEs. It means that if we use residual level as the stopping criterion for OMP, it selects a sparsity at each SNR adaptively, such that it can result in the best possible performance.

We can also see the comparison with the performance of the linear estimator which uses all subcarriers as pilots.

D. Fairness of SSS for MISO

In this subsection, we want to evaluate the fairness of SSS for the MISO channel. To have a good evaluation, we assume that individual channels between each LED and the PD are similar.

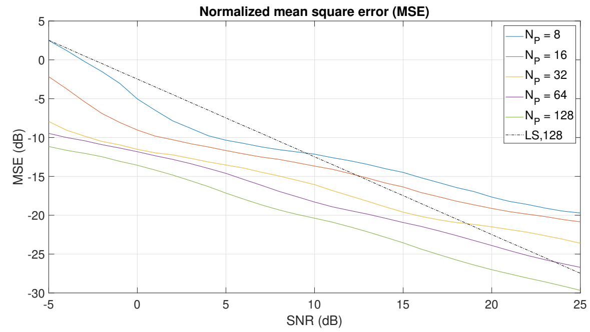

Fig. 7. The effect of the different pilot patterns on the MSE for SSS. N is 128 and $N_P = \{8, 16, 32, 64, 128\}$. The stopping criterion for OMP is the equality of the ℓ_2 norm of the residual with the noise power. The comparision is also done with the MSE of the LS estimator using the whole pilot subcarriers.

Recall that we want to have orthogonal pilots for different LEDs to be able to have interference-free channel estimation. SSS for MISO, jointly solve the problem of finding pilot patterns for all LEDs. Therefore, we can have some level of fairness.

Consider a MISO channel with 4 LEDs, in which all individual channels are random 7-sparse with the same channel impulse responses. By using SSS for MISO, we find orthogonal pilot pattern. At the PD, we utilize OMP as the estimator.

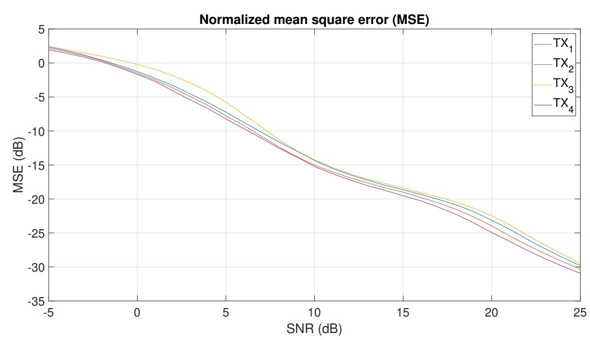

Fig. 8. Comparision of MSEs of different transmitters in a 4-MISO channel, for modified SSS. N is 128 and $N_P = 32$. The stopping criterion for OMP is the equality of the ℓ_2 norm of the residual with the noise power.

We can see the MSE for different LEDs in Figure 8 for the 4-MISO channel. By comparing them, we can see 2.5 dB difference between MSEs of different LEDs. It is also needed to say that SSS for MISO, gives 4 sets of non-overlapping pilot patterns, each with the cardinality equals 32, such that all 128 subcarriers are uses for pilots.

V. Conclusion

Although compressed sensing methods is widely studied in radio communications, it is rarely used in optical wireless communications. In this paper, we have used compressed sensing to estimate the channel in a LiFi OFDM system. We have selected SSS as an exemplary sparse pilot design algorithm. As an exemplary sparse recovery algorithm, we have chosen OMP. We have evaluated the effectiveness of these methods by means of simulations. While increasing the number of pilot subcarriers decreases the MSE, increasing the total number of subcarriers does not affect the MSE significantly. By comparing the compressed sensing based channel estimation algorithm with the linear estimator using equally-spaced pilots, we have observed that compressed sensing brings sugnificant gains for LiFi OFDM systems. The MSE decreases even though we use only few subcarriers. We have also shown that using SSS for MIMO leads to fairness between different optical wireless transmitters.

REFERENCES

[1] J. Kahn and J. Barry, "Wireless infrared communications," *Proceedings of the IEEE*, vol. 85, no. 2, pp. 265–298, 1997.

[2] Z. Gao, L. Dai, Z. Lu, C. Yuen, and Z. Wang, "Super-resolution sparse mimo-ofdm channel estimation based on spatial and temporal correlations," *IEEE Communications Letters*, vol. 18, no. 7, pp. 1266–1269, 2014.

[3] E. Candes and T. Tao, "Decoding by linear programming," *IEEE Transactions on Information Theory*, vol. 51, no. 12, pp. 4203–4215, dec 2005.

[4] D. Donoho, "Compressed sensing," *IEEE Transactions on Information Theory*, vol. 52, no. 4, pp. 1289–1306, apr 2006.

[5] G. Taubock and F. Hlawatsch, "A compressed sensing technique for OFDM channel estimation in mobile environments: Exploiting channel sparsity for reducing pilots," in *2008 IEEE International Conference on Acoustics, Speech and Signal Processing*. IEEE, mar 2008.

[6] G. Taubock, F. Hlawatsch, D. Eiwen, and H. Rauhut, "Compressive estimation of doubly selective channels in multicarrier systems: Leakage effects and sparsity-enhancing processing," *IEEE Journal of Selected Topics in Signal Processing*, vol. 4, no. 2, pp. 255–271, apr 2010.

[7] W. U. Bajwa, J. Haupt, A. M. Sayeed, and R. Nowak, "Compressed channel sensing: A new approach to estimating sparse multipath channels," *Proceedings of the IEEE*, vol. 98, no. 6, pp. 1058–1076, jun 2010.

[8] P. Pakrooh, A. Amini, and F. Marvasti, "OFDM pilot allocation for sparse channel estimation," *EURASIP Journal on Advances in Signal Processing*, vol. 2012, no. 1, mar 2012.

[9] X. He and R. Song, "Pilot pattern optimization for compressed sensing based sparse channel estimation in OFDM systems," in *2010 International Conference on Wireless Communications & Signal Processing (WCSP)*. IEEE, oct 2010.

[10] X. He, R. Song, and W.-P. Zhu, "Optimal pilot pattern design for compressed sensing-based sparse channel estimation in OFDM systems," *Circuits, Systems, and Signal Processing*, vol. 31, no. 4, pp. 1379–1395, dec 2011.

[11] C. Qi and L. Wu, "Optimized pilot placement for sparse channel estimation in OFDM systems," *IEEE Signal Processing Letters*, vol. 18, no. 12, pp. 749–752, dec 2011.

[12] ——, "A study of deterministic pilot allocation for sparse channel estimation in OFDM systems," *IEEE Communications Letters*, vol. 16, no. 5, pp. 742–744, may 2012.

[13] C. Qi, G. Yue, L. Wu, and A. Nallanathan, "Pilot design for sparse channel estimation in OFDM-based cognitive radio systems," *IEEE Transactions on Vehicular Technology*, vol. 63, no. 2, pp. 982–987, feb 2014.

[14] C. Qi, G. Yue, L. Wu, Y. Huang, and A. Nallanathan, "Pilot design schemes for sparse channel estimation in OFDM systems," *IEEE Transactions on Vehicular Technology*, vol. 64, no. 4, pp. 1493–1505, apr 2015.

[15] J.-C. Chen, C.-K. Wen, and P. Ting, "An efficient pilot design scheme for sparse channel estimation in OFDM systems," *IEEE Communications Letters*, vol. 17, no. 7, pp. 1352–1355, jul 2013.

[16] L. Najjar, "Pilot allocation by genetic algorithms forsparse channel estimation in ofdm systems," *21st European Signal Processing Conference (EUSIPCO 2013), Marrakech,, pp. 1-5.*, 2013.

[17] H. Wang, Q. Guo, G. Zhang, G. Li, and W. Xiang, "Pilot pattern optimization for sparse channel estimation in OFDM systems," *IEEE Communications Letters*, vol. 19, no. 7, pp. 1233–1236, jul 2015.

[18] J. Tropp, "Greed is good: Algorithmic results for sparse approximation," *IEEE Transactions on Information Theory*, vol. 50, no. 10, pp. 2231–2242, oct 2004.

[19] M. Elad, "Optimized projections for compressed sensing," *IEEE Transactions on Signal Processing*, vol. 55, no. 12, pp. 5695–5702, dec 2007.

[20] X. He, R. Song, and W.-P. Zhu, "Pilot allocation for sparse channel estimation in MIMO-OFDM systems," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 60, no. 9, pp. 612–616, sep 2013.

[21] ——, "Pilot allocation for distributed-compressed-sensing-based sparse channel estimation in MIMO-OFDM systems," *IEEE Transactions on Vehicular Technology*, vol. 65, no. 5, pp. 2990–3004, may 2016.

[22] R. Mohammadian, A. Amini, B. H. Khalaj, and N. Omidvar, "MIMO-OFDM pilot symbol design for sparse channel estimation," in *2015 Asia-Pacific Signal and Information Processing Association Annual Summit and Conference (APSIPA)*. IEEE, dec 2015.

[23] C. R. Berger, Z. Wang, J. Huang, and S. Zhou, "Application of compressive sensing to sparse channel estimation," *IEEE Communications Magazine*, vol. 48, no. 11, pp. 164–174, 2010.

[24] Y. Arjoune, N. Kaabouch, H. El Ghazi, and A. Tamtaoui, "Compressive sensing: Performance comparison of sparse recovery algorithms," in *2017 IEEE 7th Annual Computing and Communication Workshop and Conference (CCWC)*, 2017, pp. 1–7.

[25] D. Kanevsky, A. Carmi, L. Horesh, P. Gurfil, B. Ramabhadran, and T. N. Sainath, "Kalman filtering for compressed sensing," in *2010 13th International Conference on Information Fusion*, 2010, pp. 1–8.

[26] D. L. Donoho, Y. Tsaig, I. Drori, and J.-L. Starck, "Sparse solution of underdetermined systems of linear equations by stagewise orthogonal matching pursuit," *IEEE Transactions on Information Theory*, vol. 58, no. 2, pp. 1094–1121, feb 2012.

[27] I. Esnaola, R. E. Carrillo, J. Garcia-Frias, and K. E. Barner, "Orthogonal matching pursuit based recovery for correlated sources with partially disjoint supports," in *2010 44th Annual Conference on Information Sciences and Systems (CISS)*. IEEE, mar 2010.

[28] B. Lin, Z. Ghassemlooy, J. Xu, Q. Lai, X. Shen, and X. Tang, "Experimental demonstration of compressive sensing-based channel estimation for mimo-ofdm vlc," *IEEE Wireless Communications Letters*, vol. 9, no. 7, pp. 1027–1030, 2020.

[29] V. B. Manur and L. Ali, "Mmse based compressed sensing algorithms for channel estimation in vlc," in *2019 4th International Conference on Electrical, Electronics, Communication, Computer Technologies and Optimization Techniques (ICEECCOT)*, 2019, pp. 170–173.

[30] R. Mohammadian, A. Amini, B. H. Khalaj, and N. Omidvar, "Mimo-ofdm pilot symbol design for sparse channel estimation," in *2015 Asia-Pacific Signal and Information Processing Association Annual Summit and Conference (APSIPA)*, 2015, pp. 975–980.

[31] E. J. Candès, J. K. Romberg, and T. Tao, "Stable signal recovery from incomplete and inaccurate measurements," *Communications on Pure and Applied Mathematics*, vol. 59, no. 8, pp. 1207–1223, 2006.

[32] J. Tropp, "Greed is good: algorithmic results for sparse approximation," *IEEE Transactions on Information Theory*, vol. 50, no. 10, pp. 2231–2242, 2004.