Iterative Timing Estimation with 1-bit Quantization and Oversampling at the Receiver

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***Abstract*—Wireless communications systems of the future are expected to operate at very high bandwidths of up to multiple Gigahertz. For such systems the power consumption of analog-to-digital converters at the receiver will impose a challenge. Employing 1-bit quantization and temporal oversampling is a promising approach to increase energy efficiency. However, since 1-bit quantization is a highly non-linear operation, channel estimation and synchronization algorithms have to be revised.**

In this regard, we derive iterative data-aided timing estimators based on the expectation-maximization algorithm and the scoring algorithm assuming white noise at the receiver. Comparing the performance of the expectation-maximization algorithm based estimator, the scoring algorithm, and an existing non-data-aided least-squares estimator by numerical evaluation, we find that the scoring algorithm significantly outperforms the least squares estimator in terms of the mean square error and closely approaches the Cramér-Rao lower bound, different to the expectation-maximization based estimator whose performance stays behind the scoring algorithm. Additionally, we consider colored noise at the receiver and evaluate the influence of the timing estimation error on the system performance in terms of the spectral efficiency using zero-crossing modulation.

***Index Terms*—1-bit quantization, oversampling, timing estimation.**

I. Introduction

Demands test wireless communications systems of the future are expected to require data rates of $100\,\mathrm{Gbit/s}$ and beyond [1]. It looks promising to achieve such data rates by utilizing multi-GHz bandwidth channels in the millimeter-wave and sub-terahertz frequency bands above $100\,\mathrm{GHz}$ [2]. Due to the large signal bandwidth, the design of energy-efficient analog-to-digital converters (ADCs) is becoming a challenge. A survey by Murmann [3] compares recent ADC designs and finds that the ADC power consumption increases quadratically with the sampling rate for sampling frequencies above $300\,\mathrm{MHz}$. To compensate this, a possible solution is to reduce amplitude resolution down to a minimum, i.e., employing 1-bit quantization, as the ADC power consumption also grows exponentially with amplitude resolution. Further, 1-bit quantization comes with the advantage of reduced complexity of the analog design of the receiver, e.g., due to not requiring an automatic gain control.

With only 1-bit ADC resolution, all information needs to be conveyed in the zero-crossings of the signal. Thus, for such a system zero-crossing modulation (ZXM) [4] is a suitable modulation scheme, which can be realized employing runlength limited (RLL) sequences [5], to encode information in the temporal distances between the zero-crossings, combined with faster-than-Nyquist (FTN) signaling [6], to achieve rates beyond $1\,\mathrm{bit/s/Hz}$ per real signal dimension as proposed in [7]. A practical transceiver design with 1-bit quantization and ZXM has been presented in [8], achieving spectral efficiencies (SEs) of up to $4\,\mathrm{bit/s/Hz}$ under the assumption of perfect phase and timing synchronization. Precise timing synchronization is crucial, as with FTN signaling symbols are placed on a finer time grid. However, it is a challenging task to synchronize the transmitter and receiver, as for timing estimation only 1-bit quantized samples are available and, therefore, existing estimation approaches cannot be applied. Thus, it is necessary to revise the timing estimation and synchronization algorithms. In this regard, assuming a known timing offset, in [9] timing synchronization in a 1-bit quantized system has been studied, while in the present work we focus on timing offset estimation.

Bounds on the achievable performance of timing, phase, and frequency estimation, i.e., the Cramér-Rao lower bound (CRLB), for systems employing 1-bit quantization and oversampling at the receiver have been studied in [10]. Moreover, in [11], both a phase estimator and timing estimator have been derived, each from the least squares (LS) objective function. However, they do not achieve the theoretical possible estimation performance stated by the CRLB. For the case of phase estimation in receivers with temporally oversampled 1-bit quantization, in [12] iterative methods for maximum likelihood (ML) estimation, namely the expectation-maximization (EM) algorithm and the scoring algorithm, have been studied, which have been extended to phase noise tracking in [13]. These algorithms improve the initial LS phase estimate, which only achieves the CRLB in the low SNR regime, and close the gap to the CRLB also for mid-to-high SNRs [12]. The application and evaluation of these iterative estimation approaches for timing estimation is the main contribution of this work.

In the following, we give a short outline of this paper. In Section II, we introduce the system model. Next, we derive an EM-based estimator and the scoring algorithm for timing estimation in Section III. Moreover, in Section IV we numerically evaluate the estimation performance in terms of the mean square error (MSE) for the derived timing estimators and evaluate its effect on the SE of a system using ZXM.

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Finally, in Section V we summarize the results of this work.

Notation: In this work, vectors, matrices, random variables, and random vectors are denoted by lower case bold letters, e.g., $\mathbf{x}$, by upper case bold letters $\mathbf{X}$, by sans serif letters x, and by bold sans serif letters $\boldsymbol{\mathsf{x}}$, respectively. $\mathrm{Re}\,(z)$, $\mathrm{Im}\,(z)$, z^*, and $\arg(z)$ describe the real part, the imaginary part, the complex conjugate, and the argument of z, respectively. Also we define $j = \sqrt{-1}$. The expression $\mathbf{X}^\mathsf{T}$ denotes the transpose of $\mathbf{X}$ and $\xrightarrow{p}$ denotes convergence in probability. For $x \in \mathbb{R}$, $\lceil x \rceil = \min\{n \in \mathbb{Z} \mid n \geq x\}$. Moreover, $\mathcal{CN}$, $\mathbb{E}[\cdot]$, $\mathbb{Z}$, and $\mathbb{C}$ are the complex normal distribution, the expectation operator, the set of integers, and the set of complex numbers. When sampling a time-continuous function $x(t)$ with rate $1/T_s$, the symbol $x_k = x(kT_s)$ denotes the k-th sample.

II. System Model

We consider a system with a transmitter and a receiver which are not synchronized in time, i.e., they have an unknown deterministic timing offset. Let

$$\mathsf{u}(t) = \sum_{n=1}^{N} \mathsf{a}_n g_{\mathrm{Tx}}(t - nT - \epsilon T) \tag{1}$$

be the transmit signal constructed from a sequence of complex symbols, represented by the random vector $\boldsymbol{\mathsf{a}} = [\mathsf{a}_1, \dots, \mathsf{a}_N]^\mathsf{T}$, and the transmit filter $g_{\mathrm{Tx}}(t)$ with a single-sided bandwidth $W < \frac{1}{T}$ and symbol duration T. We model the timing offset, by shifting the transmitter time axis by ϵT with $\epsilon \in [-0.5, 0.5]$. In addition, the channel introduces a deterministic phase offset ϕ. The receiver dithers the receive signal by a known phase rotation $\varphi(t) = \Omega_{\mathrm{IF}} t$ with $\Omega_{\mathrm{IF}} \ll \frac{1}{T}$ before receive filtering and sampling at the time instants $t = k\frac{T}{M} = kT_s$ with the oversampling factor $M > 2$, resulting in the received signal

$$\mathsf{r}_k = \underbrace{\sum_{n=1}^{N} \mathsf{a}_n g(kT_s - nT - \epsilon T) e^{j(\phi + k\Omega_{\mathrm{IF}} T_s)}}_{=\mathsf{s}_k(\epsilon)} + \mathsf{n}_k. \tag{2}$$

Here, $g(t) = (g_{\mathrm{Tx}} * g_{\mathrm{Rx}})(t)$ where the receive filter $g_{\mathrm{Rx}}(t)$ has a single-sided bandwidth W_r. The channel introduces additive white Gaussian noise (AWGN) with power spectral density N_0. We express the sampled and filtered noise by the Gaussian process $\{\mathsf{n}_k\}_{k\in\mathbb{Z}}$. From the assumption $g_{\mathrm{Rx}}(t)$ being a real function, it follows that $\{\mathsf{n}_k\}_{k\in\mathbb{Z}}$ is circularly symmetric, such that the distribution of the sampled noise at the receiver is given by

$$\mathsf{n}_k \sim \mathcal{CN}(0, N_0). \tag{3}$$

In case of employing a rectangular receive filter with its bandwidth matched to the sampling rate, i.e., $W_r = \frac{1}{2T_s}$, the sampled noise at the receiver is white. Furthermore, we define the $\mathrm{SNR} = \frac{E_s}{N_0 M}$ with the symbol energy $E_s = \mathbb{E}[\mathsf{a}_n^* \mathsf{a}_n] \int_{-\infty}^{\infty} |g(t)|^2 \,\mathrm{d}t$. Finally, the samples r_k are 1-bit quantized, yielding

$$\mathsf{y}_k = \mathrm{sign}(\mathrm{Re}\,(\mathsf{r}_k)) + j\,\mathrm{sign}(\mathrm{Im}\,(\mathsf{r}_k)). \tag{4}$$

When employing 1-bit quantization the performance of the timing and phase estimation depends on the timing and phase offset of the receive signal [10]. In [10] it has been proposed to apply uniform phase and sample dithering to remove these dependencies. The same effect can be achieved by oversampling the receive signal with an irrational M at the intermediate frequency Ω_{IF} which is chosen such that $\frac{T_s \Omega_{\mathrm{IF}}}{\pi}$ is irrational. Note, that this phase dithering does not change the distribution of the circularly symmetric complex Gaussian noise.

III. Timing Estimation

In this section, we study ML timing estimation, i.e., derive estimators for the ML timing offset estimation problem

$$\hat{\epsilon} = \underset{\epsilon}{\mathrm{argmax}}\; p(\mathbf{y}|\mathbf{a};\epsilon) \tag{5}$$

where $p(\mathbf{y}|\mathbf{a};\epsilon)$ is the probability of observing the 1-bit quantized symbol vector $\mathbf{y} = [y_1, \dots, y_K]^\mathsf{T}$ with $K = \lceil MN \rceil$, given the transmit symbol sequence $\mathbf{a}$ and the timing offset ϵ. As an analytical solution of (5) is not available, we consider iterative estimation approaches. To this end, we assume the symbols in $\boldsymbol{\mathsf{a}}$ to be independent and identically distributed (i.i.d.) and zero-mean. Moreover, we employ a rectangular receive filter with bandwidth $W_r = \frac{1}{2T_s}$, i.e., we match the receive filter bandwidth to the sampling rate such that noise process $\{\mathsf{n}_k\}_{k\in\mathbb{Z}}$ at the receiver is white with variance $\sigma^2 = N_0$.

A. Expectation-Maximization Algorithm

The EM algorithm aims to find an estimate for the timing offset ϵ by maximizing $p(\mathbf{r}|\mathbf{a};\epsilon)$ instead of $p(\mathbf{y}|\mathbf{a};\epsilon)$. However, with $\mathbf{r} = [r_1, \dots, r_K]^\mathsf{T}$ being a hidden variable the EM algorithm iteratively maximizes the conditional expectation of $p(\mathbf{r}|\mathbf{a};\epsilon)$ given the observation $\mathbf{y}$. As in our case $\mathbf{y}$ is a function of $\mathbf{r}$ with the elements r_k being i.i.d. when conditioning on $\boldsymbol{\mathsf{a}}$ and ϵ, we simplify the EM algorithm as stated in [14, Chapter 7.8] to

$$\hat{\epsilon}_{l+1} = \underset{\epsilon}{\mathrm{argmax}} \sum_k \mathbb{E}_{\mathsf{r}_k}\Big[\ln p(\mathsf{r}_k|\boldsymbol{\mathsf{a}};\epsilon)\Big|\mathsf{y}_k = y_k, \boldsymbol{\mathsf{a}} = \mathbf{a}; \hat{\epsilon}_l\Big] \tag{6}$$

where $\hat{\epsilon}_l$ denotes the estimate of ϵ in the l-th iteration. Using that $\mathsf{r}_k \sim \mathcal{CN}(\mathsf{s}_k(\epsilon), \sigma^2)$ while conditioning on $\boldsymbol{\mathsf{a}}$, it follows

$$\begin{aligned}
\hat{\epsilon}_{l+1} &= \underset{\epsilon}{\mathrm{argmin}} \sum_k \mathbb{E}_{\mathsf{r}_k}\Big[|\mathsf{r}_k - \mathsf{s}_k(\epsilon)|^2\Big|\mathsf{y}_k = y_k, \boldsymbol{\mathsf{a}} = \mathbf{a}; \hat{\epsilon}_l\Big] &(7)\\
&= \underset{\epsilon}{\mathrm{argmax}} \sum_k \mathbb{E}_{\mathsf{r}_k}\Big[\mathrm{Re}\,(\mathsf{r}_k \mathsf{s}_k^*(\epsilon))\Big|\mathsf{y}_k = y_k, \boldsymbol{\mathsf{a}} = \mathbf{a}; \hat{\epsilon}_l\Big] &(8)\\
&= \underset{\epsilon}{\mathrm{argmax}} \sum_k \mathrm{Re}\Big(\mathsf{s}_k^*(\epsilon)\, \mathbb{E}_{\mathsf{r}_k}\Big[\mathsf{r}_k\Big|\mathsf{y}_k = y_k, \boldsymbol{\mathsf{a}} = \mathbf{a}; \hat{\epsilon}_l\Big]\Big) &(9)
\end{aligned}$$

where we obtain (8) using that $\sum_k |\mathsf{s}_k(\epsilon)|^2$ is independent of ϵ for large N [15, p. 391]. The conditional expectation in (9) can be expressed as

$$\mathbb{E}_{\mathsf{r}_k}\Big[\mathsf{r}_k|\mathsf{y}_k = y_k, \boldsymbol{\mathsf{a}} = \mathbf{a}; \hat{\epsilon}_l\Big] = \int_{\mathbb{C}} p(r_k|y_k, \mathbf{a}; \hat{\epsilon}_l) r_k \,\mathrm{d}r_k \tag{10}$$

$$\begin{aligned}
&= s_k(\hat{\epsilon}_l) + \frac{\sigma}{\sqrt{\pi}}\Bigg(\frac{\mathrm{Re}(y_k)}{\mathrm{erfcx}\big(-\frac{1}{\sigma}\mathrm{Re}(y_k)\mathrm{Re}(s_k(\hat{\epsilon}_l))\big)}\\
&\quad + j\,\frac{\mathrm{Im}(y_k)}{\mathrm{erfcx}\big(-\frac{1}{\sigma}\mathrm{Im}(y_k)\mathrm{Im}(s_k(\hat{\epsilon}_l))\big)}\Bigg) &(11)\\
&= f_1(y_k, \hat{\epsilon}_l) &(12)
\end{aligned}$$

where we introduce the function $f_1(y_k, \epsilon)$ as a shorthand notation and use the scaled complementary error function defined as $\mathrm{erfcx}(x) = \exp(x^2)\,\mathrm{erfc}(x) = \exp(x^2)\frac{2}{\sqrt{\pi}}\int_x^\infty e^{-t^2}\,\mathrm{d}t$.

We solve the optimization problem in (9) by utilizing the EM phase estimate [12]

$$\hat{\phi}_{l+1} = \arg\left(\sum_m u_m^*(\epsilon) e^{-j\varphi_m} f_1(y_m, \hat{\epsilon}_l)\right) \tag{13}$$

where $u_k(\epsilon) = u(kT_s)$ with $u(t)$ given in (1) and $\varphi_k = kT_s\Omega_{\mathrm{IF}}$ is due to phase dithering, such that

$$\begin{aligned}
\hat{\epsilon}_{l+1} &= \underset{\epsilon}{\operatorname{argmax}}\, \mathrm{Re}\Bigg(\sum_k u_k^*(\epsilon) e^{-j\varphi_k} f_1(y_k, \hat{\epsilon}_l) \\
&\qquad \times \exp\left(-j\arg\left(\sum_m u_m^*(\epsilon) e^{-j\varphi_m} f_1(y_m, \hat{\epsilon}_l)\right)\right)\Bigg) &(14)\\
&= \underset{\epsilon}{\operatorname{argmax}} \left|\sum_k u_k^*(\epsilon) e^{-j\varphi_k} f_1(y_k, \hat{\epsilon}_l)\right| &(15)\\
&= \underset{\epsilon}{\operatorname{argmax}} \left|\sum_k u_k^*(\epsilon) e^{-j\varphi_k} f_1(y_k, \hat{\epsilon}_l)\right|^2 &(16)\\
&\xrightarrow{p} \underset{\epsilon}{\operatorname{argmax}} \sum_k\sum_m \left(f_1(y_k, \hat{\epsilon}_l) e^{-j\varphi_k}\right)^* \left(f_1(y_m, \hat{\epsilon}_l) e^{-j\varphi_m}\right) \\
&\qquad \times \sum_{n=1}^{N} g(kT_s - nT - \epsilon T) g(mT_s - nT - \epsilon T). &(17)
\end{aligned}$$

We obtain (15) with $|z| = z\, e^{-j\arg(z)}$ and (17) is due to the law of large numbers and the fact that the symbols a_n are zero-mean i.i.d., where we also drop the constant $\mathbb{E}\left[|\mathsf{a}_n|^2\right]$. Also, $g_{\mathrm{Tx}}(t) = g(t)$, assuming the receive filter being a rectangular filter with $W_r = \frac{1}{T_s}$. As described in [15, Chapter 7.6.1], the inner sum in (17) is periodic in ϵT and as $g(t)$ is bandlimited to $\frac{1}{T}$ and real valued it can be represented by the first two terms of its Fourier series. Using this, we solve for $\hat{\epsilon}_{l+1}$ yielding

$$\begin{aligned}
\hat{\epsilon}_{l+1} = -\frac{1}{2\pi}\arg\Bigg(&\sum_k\sum_m \left(f_1(y_k, \hat{\epsilon}_l) e^{-j\varphi_k}\right)^* \\
&\times \left(f_1(y_m, \hat{\epsilon}_l) e^{-j\varphi_m}\right) q((k-m)T_s) e^{-j\pi\frac{k+m}{M}}\Bigg) \quad (18)
\end{aligned}$$

with $q(t)$ being the inverse Fourier transform of

$$Q(f) = G\left(f - \frac{1}{2T}\right) G^*\left(f + \frac{1}{2T}\right) \tag{19}$$

and $G(f)$ being the Fourier transform of the combined transmit and receive filter $g(t)$.

B. Scoring Algorithm

The Fisher scoring algorithm is an iterative estimation approach, which finds the maximum of the log-likelihood function, similar to the Newton-Raphson method, based on first and second order derivatives of the objective function. Provided with an initial estimate $\hat{\epsilon}_0$, the estimate of the Fisher scoring algorithm can be iteratively calculated using [14, Section 7.7]

$$\hat{\epsilon}_{l+1} = \hat{\epsilon}_l + \mathcal{I}^{-1}(\hat{\epsilon}_l) V(\hat{\epsilon}_l) \tag{20}$$

where $\hat{\epsilon}_l$ is the estimate of the l-th iteration. Further, $V(\epsilon)$ is the score function, i.e., first derivative of the log-likelihood function, and $\mathcal{I}(\epsilon)$ the Fisher information (FI), i.e., the negative expectation of the second derivative of the log-likelihood function. Under certain regularity conditions [16] the estimate $\hat{\epsilon}_l$ approaches the ML estimate as $l \to \infty$.

While an analytical expression for the FI $\mathcal{I}(\epsilon)$ is not known, for $\{\mathsf{n}_k\}_{k\in\mathbb{Z}}$ being white noise and the symbols a_n being i.i.d., it has been shown in [10] that a lower bound on $\mathcal{I}^{-1}(\epsilon)$ is given by

$$\begin{aligned}
\mathcal{I}^{-1}(\epsilon) &\geq (\mathcal{I}_{\mathrm{UB}})^{-1} &(21)\\
&= \left(4\pi N T^2 \frac{E_s}{N_0} \frac{\int_{-\infty}^{\infty} f^2 |G(f)|^2\,\mathrm{d}f}{\int_{-\infty}^{\infty} |G(f)|^2\,\mathrm{d}f} \kappa_1\left(\frac{E_s}{N_0 M}\right)\right)^{-1} &(22)\\
\kappa_1(x) &= c_1 e^{-c_2 x}(I_0(c_2 x) + I_1(c_2 x)) &(23)
\end{aligned}$$

where the constants are given by $c_1 = 4.0360$, $c_2 = 0.3930$, and $I_\nu(\cdot)$ is the modified Bessel function of the first kind of order ν. The application of this lower bound instead of the exact expression for the inverse of the FI in (20) is possible, as the lower-bounding only has an impact on the convergence speed but does not change the limit, i.e., the value of $\lim_{l\to\infty}\hat{\epsilon}_l$ of the scoring algorithm.

Under the assumption of white noise samples, the score function is obtained by taking the derivative of the log-likelihood function yielding

$$\begin{aligned}
V(\epsilon) &= \frac{\partial}{\partial\epsilon}\sum_{k=1}^{K} \ln p(y_k|\mathbf{a};\epsilon) &(24)\\
&= \frac{\partial}{\partial\epsilon}\sum_{k=1}^{K}\Bigg(\ln\left(\frac{1}{2}\mathrm{erfc}\left(-\frac{1}{\sigma}\mathrm{Re}\,(y_k)\,\mathrm{Re}\,(s_k(\epsilon))\right)\right) \\
&\qquad + \ln\left(\frac{1}{2}\mathrm{erfc}\left(-\frac{1}{\sigma}\mathrm{Im}\,(y_k)\,\mathrm{Im}\,(s_k(\epsilon))\right)\right)\Bigg) &(25)\\
&= \sum_{k=1}^{K}\mathrm{Re}\Big(\frac{\partial s_k^*(\epsilon)}{\partial\epsilon} f_2(y_k, \epsilon)\Big) &(26)
\end{aligned}$$

where the function f_2 is given by

$$\begin{aligned}
f_2(y_k, \epsilon) = \frac{2}{\sigma\sqrt{\pi}}\Bigg(&\frac{\mathrm{Re}\,(y_k)}{\mathrm{erfcx}(-\frac{1}{\sigma}\mathrm{Re}\,(y_k)\,\mathrm{Re}\,(s_k(\epsilon)))} \\
&+ j\,\frac{\mathrm{Im}\,(y_k)}{\mathrm{erfcx}(-\frac{1}{\sigma}\mathrm{Im}\,(y_k)\,\mathrm{Im}\,(s_k(\epsilon)))}\Bigg). \quad (27)
\end{aligned}$$

Finally, for practical reasons, we state the solution as a matched filter operation

$$V(\epsilon) = \mathrm{Re}\left(e^{-j\phi}\sum_{n=1}^{N} a_n^* \dot{z}_2(nT + \epsilon T, \epsilon)\right) \tag{28}$$

with

$$\begin{aligned}
\dot{z}_2(t, \epsilon) &= (h_d * z_2)(t, \epsilon) &(29)\\
&= \sum_{k=-\infty}^{\infty} (h_d * g_{\mathrm{MF}})(t - kT_s) f_2(y_k, \epsilon)\, e^{-j\varphi_k} &(30)
\end{aligned}$$

where $g_{\rm MF}(t) = g(-t)$ is the matched filter, $z_2(t,\epsilon)$ the matched filter output, and h_d the digital differentiator [17, Chapter 5.6.1] defined by $h_d(kT_s) = 0$ for $k = 0$ and $h_d(kT_s) = \frac{1}{kT_s}(-1)^k$ for $k \neq 0$.

C. Discussion

Both the EM-based approach and the scoring algorithm require an initial value $\hat{\epsilon}_0$ for the recursion in (18) and (20). For initialization, we utilize the LS-based estimator [11], i.e.,

$$\hat{\epsilon}_0 = \hat{\epsilon}_{\rm LS} = -\frac{1}{2\pi}\arg\left(\sum_k\sum_m \left(y_k e^{-j\varphi_k}\right)^* \left(y_m e^{-j\varphi_m}\right) \times q((k-m)T_s) e^{-j\pi\frac{k+m}{M}}\right) \quad (31)$$

where $q(t)$ is the inverse Fourier transform of (19), and which is similar to the timing estimator for the unquantized case given in [15, Section 8.4].

Note that both, the EM-based algorithm and the scoring algorithm, require knowledge of the transmit symbols $\mathbf{a}$ and of the phase offset ϕ, i.e., they are data-aided estimators, whereas the LS estimator in (31) is a non-data-aided estimator. This means that different to the LS estimator the iterative estimators require pilot symbols and in addition a prior phase estimate, which can be obtained after an initial LS-based timing estimate [11].

We derived the estimators under the assumption of white noise at the receiver. However, the power of the noise samples can be reduced by matching the bandwidth of the receive filter to the transmit signal bandwidth with $W_r = W + \frac{\Omega_{\rm IF}}{2\pi}$. In this case the noise samples are correlated, i.e., we have colored noise at the receiver. Deriving an estimator for the case of colored noise has not been possible, as there is no general expression for orthant probabilities [18]. However, in the following we evaluate the estimation performance of the derived estimators also with colored noise, corresponding to mismatched estimators, as performance gains have been observed in the similar case of phase estimation in [12].

D. Consistency

In the following, we analyze if the iterative estimators derived in this work are consistent estimators if they are initialized by $\hat{\epsilon}_0 = \hat{\epsilon}_{\rm LS}$, which is a consistent estimate [11].

For the scoring algorithm it can be shown that the estimates $\hat{\epsilon}_l$ are consistent if the scoring algorithm is initialized by a consistent estimate, i.e., it holds that

$$\hat{\epsilon}_l \xrightarrow{p} \epsilon \text{ as } N \to \infty. \quad (32)$$

In the following, we provide a sketch of the proof. For the update step $\Delta_l = \mathcal{I}_{\rm UB}^{-1}\, V(\hat{\epsilon}_l)$ we can show that

$$\Delta_l \xrightarrow{p} \mathcal{I}_{\rm UB}^{-1}\, V(\epsilon) = 0 \quad (33)$$

with the help of the continuous mapping theorem [19]. Moreover, in (33) we utilize the fact that the lower bound on the inverse of the FI $\mathcal{I}_{\rm UB}^{-1}$ is a positive constant w.r.t. the parameter ϵ and that the score function $V(\epsilon)$ is zero at true timing offset ϵ, as the log-likelihood function is maximized for the true timing offset ϵ. Applying (33) to the RHS of (20) yields the induction step

$$\hat{\epsilon}_l \xrightarrow{p} \epsilon \Rightarrow \hat{\epsilon}_{l+1} \xrightarrow{p} \epsilon. \quad (34)$$

By induction, it follows that if $\hat{\epsilon}_0$ is consistent, all estimates $\hat{\epsilon}_l$ are consistent as stated in (32). With the scoring algorithm being a consistent estimator, it is also asymptotically unbiased, i.e., $\mathbb{E}\left[\hat{\epsilon}_l\right] \xrightarrow{p} \epsilon$.

In general, the EM algorithm converges under mild assumptions to the ML estimate [20] and, therefore, provides consistent estimates. However, in contrast to the scoring algorithm, our numerical evaluations show that the EM-based estimator in (18) does not converge towards the ML estimate. The reason is that in (8) and (17) we consider $N \to \infty$, i.e., infinitely many observations. However, the numerical evaluation in the following section shows that for the considered case of $N = 100$ the estimator given by (18) does not converge to the ML estimate, such that for finite N the given EM-based estimator is not consistent.

IV. Performance Evaluation

We evaluate the performance of the presented timing estimators in terms of their MSE. Moreover, to evaluate if the achieved timing estimation performance is sufficient, we study the influence of timing estimation error on the system performance in terms of the SE for a system using ZXM.

A. Performance of Timing Estimation

We compare the performance of the timing estimators in terms of the $\mathrm{MSE}(\hat{\epsilon}) = \mathbb{E}\left[(\hat{\epsilon}-\epsilon)^2\right]$. For this purpose, we consider a pilot sequence consisting of $N = 100$ i.i.d. QPSK symbols. For the transmit filter we consider a root-raised-cosine (RRC) transmit filter with a roll-off factor $\alpha = 0.6$, cf. [8]. At the receiver, we apply a frequency offset $\Omega_{\rm IF} = 0.12\pi$ and sample with different oversampling factors M. For initialization of the iterative estimators we utilize the LS estimator (31) and we stop after a fixed number of 50 iterations for the EM-based as well as the scoring algorithm, where both iterative algorithms have converged, which is not displayed here. Comparing the EM-based estimator and the scoring algorithm in terms of convergence speed, numerical evaluations show that the scoring algorithm converges substantially faster that the EM-based algorithm and shows no significant changes after 4 iterations.

First, we evaluate the performance of the estimators for the white noise setting as considered for their derivation, i.e., $W_r = \frac{1}{2T_s}$. The results for the estimation performance over E_s/N_0 are displayed in Fig. 1. We observe that the scoring algorithm is able to greatly improve the initial LS-estimate and achieves the lowest MSE of all estimators, while also being very close to the lower bound on the CRLB (CRLB-LB) given by (22). At mid-to-high SNR, oversampling has a positive effect on the performance, which diminishes for lower SNR. The EM-based estimator however has a higher MSE than expected. It does not converge toward the ML estimate in general and performs even worse than the LS estimator at high SNR. This is likely due to the fact that for numerical evaluation we

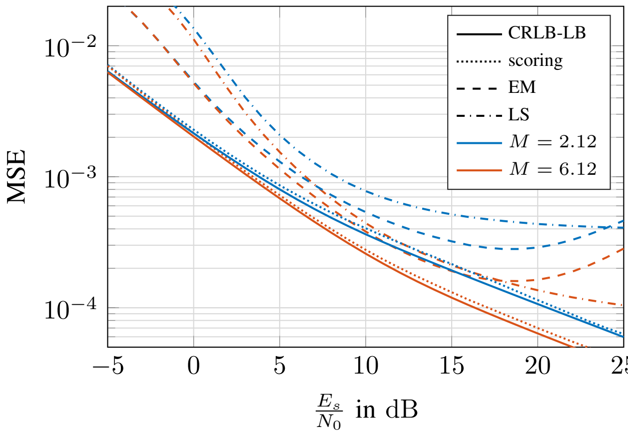

Fig. 1. MSE over E_s/N_0 of timing estimators when employing a rectangular receive filter with $W_r = 1/(2T_s)$

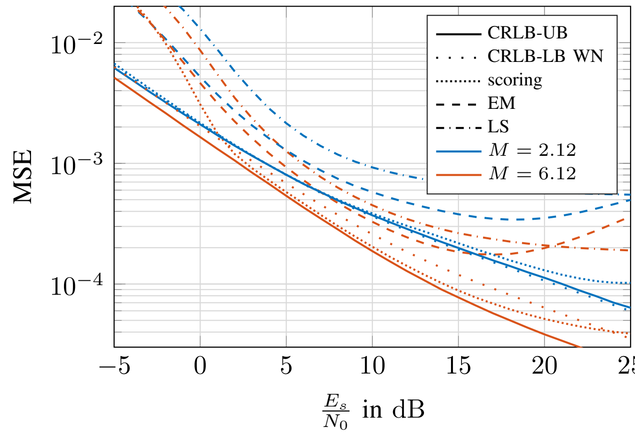

Fig. 2. MSE over E_s/N_0 of timing estimators when employing an RRC receive filter with $W_r = W + \Omega_{\mathrm{IF}}/(2\pi)$; both CRLBs for a rectangular receive filter, CRLB-LB for white noise (WN) with $W_r = 1/(2T_s)$

consider a finite number of $N = 100$ pilot symbols, while for the derivation of the EM-based estimator we assume $N \to \infty$ in (8) and (17).

In case of adapting the receive filter bandwidth to the transmit signal bandwidth, i.e., $W_r = W + \frac{\Omega_{\mathrm{IF}}}{2\pi}$, we evaluate the estimators introduced in this work in a mismatched setting. The estimation performance for this case is displayed in Fig. 2. As no closed-form CRLB for timing estimation with colored noise is known, we compare the MSE to an upper bound on the CRLB (CRLB-UB) [21] [10] computed by Monte Carlo simulation. We observe a similar behavior as for white noise, the scoring algorithm outperforms the EM- and LS-based estimation approaches and also is very close the CRLB-UB except for high SNR and for $M = 6.12$ in the low SNR regime. In case of a low SNR and $M = 6.12$ the effect of the noise correlation not considered in the derivation of the estimators increases and leads to a decreased estimation performance. Moreover, also in the high SNR regime the scoring algorithm deviates from the CRLB-UB due to the mismatched estimation approach. Furthermore, for $M = 6.12$ for a wide SNR range the MSE of the scoring algorithm is lower than the CRLB for the white noise case. In this range the advantage of a smaller noise power due to a reduced receive filter bandwidth is larger than the effect of the mismatched estimation. I.e., for higher oversampling factors the estimation performance benefits from lowering the bandwidth of the receive filter. This also coincides with the fact that for $M = 6.12$ the CRLB-UB for colored noise is lower than the CRLB-LB for the white noise case. We conclude that it is beneficial to match the bandwidth of the receive filter to the signal bandwidth for a large range of SNRs.

B. Communication Performance

To assess if the achieved timing estimation performance is sufficient, we evaluate the communication performance in terms of spectral efficiency when using ZXM in the presence of the remaining timing estimation error. For this purpose, we consider the transceiver design including RLL encoding and corresponding receiver side demapping presented in [8]. The system employs ZXM using RLL sequences in combination with FTN signaling, where the number of transmit symbols per Nyquist interval T is denoted by M_{Tx}. At the receiver, the signal is sampled with rate $\frac{1}{T_s} = \frac{M M_{\mathrm{Tx}}}{T}$ and 1-bit quantized. In the present work, additionally to [8] we consider a timing offset of the receiver and its impact on the system performance. To this aim, we shift the sample time instants of the received signal by $\epsilon_\Delta = \epsilon - \hat{\epsilon}$, which is the remaining timing offset after a timing synchronization by adapting the sampling time instants based on the estimate provided by the considered timing estimation algorithm, i.e., an analog timing synchronization.

To obtain an estimate for the SE, we follow the same approach as described in [8] based on lower-bounding the mutual information (MI) between a block of p input bits c_i of the RLL encoder and the corresponding log-likelihood ratios (LLRs) λ_i at the output of the RLL decoder by

$$I(\mathbf{c};\boldsymbol{\lambda}) \geq \sum_{i=1}^{p} I(\mathsf{c}_i;\lambda_i). \tag{35}$$

Using (35), the SE is lower-bounded as

$$\mathrm{SE}_{\mathrm{LB}} = \frac{M_{\mathrm{Tx}} R_{\mathrm{RLL}}}{W_{95\%} T} \frac{1}{p} \sum_{i=1}^{p} I(\mathsf{c}_i;\lambda_i) \tag{36}$$

where R_{RLL} is the code rate of the RLL code. Moreover, we consider the fractional power containment bandwidth $W_{95\%}$, allowing 5% out-of-band (OOB) emission. Note that, when calculating the SNR, we use [8, Eq. (61)] to obtain the symbol energy E_s of the RLL sequences. We refer the reader to [8] and [22] for a more detailed description of the evaluation method.

In Fig. 3 we compare the SE of a system with perfect timing synchronization ($\epsilon_\Delta = 0$) and a system with a timing offset after employing LS and scoring algorithm based timing estimators. Moreover, the AWGN capacity $\mathrm{SE}_{\mathrm{AWGN}} = \frac{1}{95\%} \log_2(1+\mathrm{SNR})$ is displayed. The EM-based approach is omitted here, due to its subpar estimation performance.

In general, we find that the superior estimation performance of the scoring algorithm compared to the LS estimator translates to a performance increase w.r.t. SE. Without FTN signaling, i.e., $M_{\mathrm{Tx}} = 1$, we observe only marginal increases in SE at low SNR when utilizing an estimate obtained by the scoring

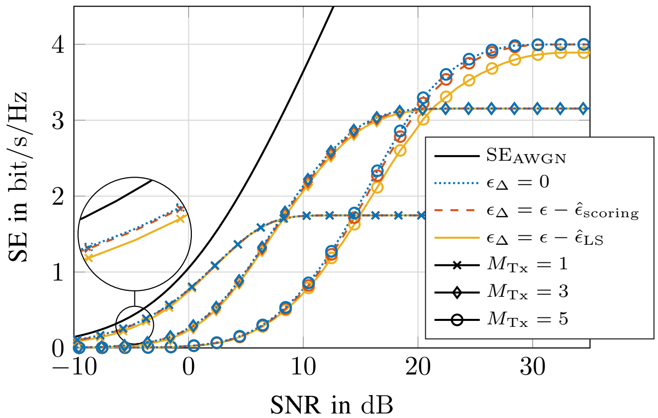

Fig. 3. Lower bound on SE over SNR depending on timing estimation approach for different FTN signaling factors M_{Tx}, oversampling factor $M = 2$, and $W_r = W + \Omega_{\mathrm{IF}}/(2\pi)$

algorithm compared to the LS-based estimator. In this domain it is sufficient to rely on estimates of the LS method, which has a lower computational complexity. With increasing M_{Tx} the performance gains by using the scoring algorithm instead of the LS timing estimator increase, where for $M_{\mathrm{Tx}} = 5$ the SE of a system using the scoring algorithm based timing estimator is substantially higher than in the case of using the LS timing estimator, greatly reducing the performance gap to the case of perfect timing synchronization. The reason is that with higher FTN signaling factors, the zero-crossings are placed on a finer timing grid, such that the system is more sensitive to timing errors.

V. Conclusion

In this work, we studied iterative data-aided timing estimation algorithms for systems employing 1-bit quantization and temporal oversampling at the receiver. We derive such estimators based on the expectation-maximization algorithm and the scoring algorithm, both under the assumption of white noise at the receiver. Considering white noise, our numerical evaluations show that the scoring algorithm closely approaches the CRLB and achieves the lowest MSE compared to the EM-based approach and the existing LS timing estimator, where however the LS estimator is non-data-aided and has a lower computational complexity. Also, we observe that the estimation performance of the scoring algorithm for high oversampling factors in a wide SNR range benefits from matching the bandwidth of the receive filter to the signal bandwidth, which results in colored noise and, thus, mismatched estimation. However, at low and high SNR this mismatch leads to a performance degradation.

Furthermore, we evaluate the communication performance for a system using ZXM and an analog synchronization based on the different timing estimation algorithms, showing that a system utilizing LS timing estimation does not achieve the theoretically possible system performance at high FTN signaling factors, while the scoring algorithm is able to almost close this gap.

Overall, the presented scoring algorithm enables a sufficiently precise timing estimation and, thus, is a further major step towards the realization of energy-efficient wideband millimeter-wave and sub-terahertz communications systems using ZXM with high FTN signaling factors and receivers with 1-bit quantization.

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