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Robust Sum Rate Maximization in the Multi-Cell MU-MIMO Downlink

Richard Fritzsche, Gerhard P. Fettweis
Technische Universität Dresden, Vodafone Chair Mobile Communications Systems, Dresden, Germany
Email: {richard.fritzsche, fettweis}@ifn.et.tu-dresden.de

***Abstract*—This paper studies linear precoding designed for the multi-cell multi-user multiple-input–multiple-output (MU-MIMO) downlink. The objective is to maximize the weighted sum rate (WSR) under imperfect channel state information (CSI) conditions and per base station (BS) transmit power constraints. The expectation of the WSR over the CSI error can be lower bounded by minimizing the expected weighted sum mean square error (MSE) assuming minimum MSE (MMSE) receive filters. The problem can be solved by an iterative algorithm which alternately calculates the MMSE receive filters given a fixed precoding matrix and vice versa. The algorithm converges to a local optimum. For the optimization of the precoding matrix under per BS power constraints, we present two robust solutions. The first one is based on the transmit Wiener filter solution under a sum power constraint combined with a consistent scaling to satisfy each per BS power constraint. In the second solution the problem is transformed into a second order cone program (SOCP) where per BS power constraints can be directly included. Simulation results show performance gains compared to robust and non-robust state of the art schemes.**

I. Introduction

Joint processing in cooperative cellular networks theoretically provides substantial performance gains compared to non-cooperative systems, especially for users located in cell edge areas [1]. Multiple antenna base stations (BSs) together with multi-antenna user equipments (UEs) constitute a virtual multi-user multiple-input–multiple-output (MU-MIMO) system, also referred to as network MIMO [2]. In the downlink, interference between UEs is handled already at the transmitter side, as long as the degrees of freedom at the UE side (number of receive antennas) is smaller than the number of superimposed data streams. Interference mitigation at the transmitter can be implemented by means of precoding. While capacity in the MU-MIMO downlink can be achieved with non-linear precoding using dirty paper coding [3], implementing non-linear techniques ([4], [5]) suffer from high complexity and makes linear precoding attractive for practical systems [6], [7]. The aforementioned precoding strategies are based on the assumption that perfect channel state information (CSI) is available at the transmitter side. However, in practice CSI is typically only imperfectly available due to impairments like, e.g., channel estimation errors, feedback compression or delays. Imperfections in the CSI can lead to substantial performance degradation [8]. In order to increase robustness against impaired CSI, statistical knowledge of the CSI imperfections can be incorporated into the precoder design [9], [10].

In this paper, we address the problem of maximizing the weighted sum rate (WSR) under per BS power constraints and CSI imperfections. The WSR maximization under a sum power constraint and perfect CSI conditions was addressed in [11] by introducing additional zeros forcing constraints. In [12] it was found that the WSR maximization problem can be solved by means of the weighted sum mean square error (MSE) minimization assuming that minimum MSE (MMSE) receive filters are applied at the UEs. The authors proposed a suboptimal iterative algorithm, where the precoding matrix is optimized for given receive filters resulting from the precoding matrix of the previous iteration. The alternating algorithm converges to a local optimum. In [13] this approach was extended to multi-cell precoding with per BS power constraints. A solution for the WSR maximization problem under sum power constraints and imperfect CSI conditions was presented in [14] where the expectation of the WSR over the CSI uncertainty was lower bounded by using the expectation over the MSE matrix, which makes the problem mathematically tractable. However, the presented solution assumes that user data is not shared between BSs.

In this contribution, we first extend the algorithm of [12] by using the approximation of [14]. We present a robust precoder design optimized for a sum power constraint. In contrast to [14], our proposed solution takes into account that the CSI error variance can be different for each BS-UE link. The solution can be applied to multi-cell setups by consistently scaling the precoding matrix in order to satisfy each per BS power constraint. Secondly, we present a robust design which directly optimizes the precoding matrix under per BS power constraints by adapting the solution of [10].

The remainder of this paper is structured in the following way. The system model is introduced in Section II. Section III presents filter optimization for single-cell setups, while in Section IV multi-cell setups are addressed. Section V shows simulation results followed by conclusions in Section VI.

Notation: Conjugate, transposition and conjugate transposition is denoted with $(\cdot)^*$, $(\cdot)^T$ and $(\cdot)^H$, respectively. The trace of a matrix is written as $\mathrm{tr}(\cdot)$, $\det(\cdot)$ denotes determinant, while $||\cdot||$ is used for Frobenius norm. The operator $\mathrm{dg}(\cdot)$ replaces each non-diagonal element of a matrix with zero. Expectation is $\mathbb{E}\{\cdot\}$. The matrix operator $\odot$ refers to element wise multiplication. $\mathbb{C}$ donates the set of complex numbers and $\mathcal{N}_{\mathbb{C}}(\mathbf{m}, \mathbf{\Phi})$ refers to a multi-variate complex normal distribution with mean vector $\mathbf{m}$ and covariance matrix $\mathbf{\Phi}$.

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II. System Model

We consider a network MIMO system with M BSs jointly transmitting to K UEs. The set of BSs and UEs is denoted by $\mathcal{M} = \{1, \ldots, M\}$ and $\mathcal{K} = \{1, \ldots, K\}$, respectively. Each BS m is equipped with B_m transmit antennas while each UE k employs U_k receive antennas. The overall number of antennas at the BS and UE side is B and U, respectively. The data vector $\mathbf{d} = [\mathbf{d}_1^T, \ldots, \mathbf{d}_K^T]^T \sim \mathcal{N}_\mathbb{C}(\mathbf{0}, \mathbf{I})$ is jointly precoded at all M BSs using the precoding matrix $\mathbf{B} = [\mathbf{B}_1, \ldots, \mathbf{B}_K]$. $\mathbf{B}_k \in \mathbb{C}^{[B \times U_k]}$ is the partition of $\mathbf{B}$ applied at all M BSs in order to precode the data of UE k. Each BS m needs to restrict its transmit power to $\mathrm{tr}\{\mathbf{S}_m \mathbf{B}\mathbf{B}^H\} \leq \rho_m$. In order to select the partition of $\mathbf{B}$ which is applied at BS m the diagonal matrix $\mathbf{S}_m$ carries ones on the diagonal elements corresponding to the transmit antennas of BS m and zeros otherwise. The precoded symbol vector $\mathbf{x} = \mathbf{B}\mathbf{d}$ is transmitted over a frequency flat complex Gaussian distributed channel denoted as $\mathbf{H} = [\mathbf{H}_1^T, \ldots, \mathbf{H}_K^T]^T$. Matrix $\mathbf{H}_k = [\mathbf{H}_{k,1}, \ldots, \mathbf{H}_{k,M}]$ is the channel to UE k and $\mathbf{H}_{k,m} \in \mathbb{C}^{[U_k \times B_m]}$ is the channel from BS m to UE k. It is assumed that the entries of $\mathbf{H}$ are uncorrelated and the elements of $\mathbf{H}_{k,m}$ are independent and identically distributed (i.i.d.) according to $\mathrm{vec}(\mathbf{H}_{k,m}) \sim \mathcal{N}_\mathbb{C}(\mathbf{0}, \lambda_{k,m}\mathbf{I})$, $\forall k, m$, while $\mathrm{vec}(\cdot)$ stacks the columns of a matrix into a vector. The mean channel gain of each link between BS m and UE k is

$$\lambda_{k,m} = \beta d_{k,m}^{-\alpha} \tag{1}$$

with path loss exponent α, distance $d_{k,m}$ between UE k and BS m and coefficient β to further adjust the model. Additionally, it is assumed that the channel remains constant over the duration of a data block. The received signal vector at UE k is impaired by additive white Gaussian noise (AWGN) denoted as $\mathbf{n}_k \sim \mathcal{N}_\mathbb{C}\left(\mathbf{0}, \sigma_n^2\mathbf{I}\right)$ before it is equalized using the linear receive filter $\mathbf{U}_k$. The transmission equation is obtained by stacking the equalized data symbols of all K UEs into a single vector

$$\hat{\mathbf{d}} = \mathbf{U}\left(\mathbf{H}\mathbf{B}\mathbf{d} + \mathbf{n}\right) = \mathbf{U}[\mathbf{y}_1^T, ..., \mathbf{y}_K^T]^T. \tag{2}$$

The receive filters of all UEs are collected in matrix $\mathbf{U} = \mathrm{blkdiag}\left(\mathbf{U}_1, \ldots, \mathbf{U}_K\right)$ while $\mathbf{n} = [\mathbf{n}_1^H, \ldots, \mathbf{n}_K^H]^H$ is the overall noise vector. The operator $\mathrm{blkdiag}(\cdot)$ constructs a block diagonal matrix.

In this work, we assume that the precoding matrix $\mathbf{B}$ is computed at a central unit (CU), which has access to the data of all UEs. Furthermore, the CU is connected to all BSs via the backhaul. In order to compute the precoder, CSI of all users (i.e., the complete channel matrix $\mathbf{H}$) needs to be available at the CU. Therefore, it is assumed that the UEs observe their channel based on pilot signals, which are a priori known at the UEs. Since uplink resources are limited the channel observations are compressed for feedback transmission. The CSI of the overall channel available at the CU after feedback transmission is denoted as $\hat{\mathbf{H}}$. For the relation between the actual channel $\mathbf{H}$ and the available CSI $\hat{\mathbf{H}}$ we are using the feedback model stated in Sec. II B of [10]. Due to the CSI impairments resulting from feedback transmission the actual channel known at the CU can be interpreted as a random variable

$$\mathbf{H} = \hat{\mathbf{H}} + \mathbf{E} \tag{3}$$

with mean $\hat{\mathbf{H}}$ and the zero mean random error matrix $\mathbf{E} = [\mathbf{E}_1^T, \ldots, \mathbf{E}_K^T]^T$ with $\mathbf{E}_k = [\mathbf{E}_{k,1}, \ldots, \mathbf{E}_{k,M}]$. Corresponding to the properties of the channel matrix the elements of $\mathbf{E}$ are uncorrelated and the elements of $\mathbf{E}_{k,m}$ are i.i.d. with $\mathrm{vec}(\mathbf{E}_{k,m}) \sim \mathcal{N}_\mathbb{C}(\mathbf{0}, \sigma_{k,m}^2\mathbf{I})$, $\forall k, m$. The motivation for distinct error variances for each BS-UE channel results from separate path losses.

While the precoding matrix $\mathbf{B}$ is calculated at the CU based on impaired CSI, each receive filter $\mathbf{U}_k$ is computed at its UE k based on precoded pilots, which are assumed to be perfectly available at the receiver side.

A. Objective

We are targeting the maximization of the WSR under per BS power constraints by properly choosing the precoding matrix $\mathbf{B}$. This problem can be written as

$$\begin{aligned} \mathbf{B}^\star = \quad & \underset{\mathbf{B}}{\mathrm{argmax}} \quad \textstyle\sum_{k=1}^K \nu_k R_k \\ & \quad \text{s.t.} \quad \mathrm{tr}(\mathbf{S}_m\mathbf{B}\mathbf{B}^H) \leq \rho_m \quad \forall m. \end{aligned} \tag{4}$$

The weights $\nu_k \geq 0$ can be adapted in order to apply a certain priorization. The achievable rate of each user k reads

$$R_k = \log\det\left(\mathbf{I} + \mathbf{B}_k^H\mathbf{H}_k^H\mathbf{C}_k^{-1}\mathbf{H}_k\mathbf{B}_k\right), \tag{5}$$

where inter-user interference and noise available at user k is gathered in the covariance matrix

$$\mathbf{C}_k = \sigma_n^2\mathbf{I} + \sum_{l=1, l\neq k}^{K} \mathbf{H}_k\mathbf{B}_l\mathbf{B}_l^H\mathbf{H}_k^H. \tag{6}$$

In the following sections we exploit a basic relation between achievable rate and MSE. For that purpose we introduce the MSE covariance matrix between the actual data symbol vector of user k and its estimate

$$\begin{aligned} \mathbf{M}_k = \; & \mathbb{E}\left\{(\mathbf{d}_k - \mathbf{U}_k\mathbf{y}_k)(\mathbf{d}_k - \mathbf{U}_k\mathbf{y}_k)^H\right\} \\ = \; & \mathbf{I} + \mathbf{U}_k\mathbf{H}_k\mathbf{B}\mathbf{B}^H\mathbf{H}_k^H\mathbf{U}_k^H + \sigma_n^2\mathbf{U}_k\mathbf{U}_k^H \\ & -\mathbf{U}_k\mathbf{H}_k\mathbf{B}_k - \mathbf{B}_k^H\mathbf{H}_k^H\mathbf{U}_k^H. \end{aligned} \tag{7}$$

The receive filter which minimizes $\mathrm{tr}(\mathbf{M}_k)$ results in

$$\mathbf{U}_k^{\mathrm{MMSE}} = \mathbf{B}_k^H\mathbf{H}_k^H(\mathbf{H}_k\mathbf{B}\mathbf{B}^H\mathbf{H}_k^H + \sigma_n^2\mathbf{I})^{-1}. \tag{8}$$

The MMSE matrix of user k is obtained by inserting the MMSE receive filters (8) into (7)

$$\begin{aligned} \mathbf{M}_k^{\mathrm{MMSE}} & = \mathbb{E}\left\{(\mathbf{d}_k - \mathbf{U}_k^{\mathrm{MMSE}}\mathbf{y}_k)(\mathbf{d}_k - \mathbf{U}_k^{\mathrm{MMSE}}\mathbf{y}_k)^H\right\} \\ & = (\mathbf{I} + \mathbf{B}_k^H\mathbf{H}_k^H\mathbf{C}_k^{-1}\mathbf{H}_k\mathbf{B}_k)^{-1}. \end{aligned} \tag{9}$$

Hence, the rate in (5) can equivalently be expressed as

$$R_k = -\log\det\left(\mathbf{M}_k^{\mathrm{MMSE}}\right), \tag{10}$$

leading to the algorithms of the following sections.

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III. Single-Cell Optimization

A solution of problem (4) for the special case of a sum power constraint ($M = 1$) was stated in [12]. It was found that for a given precoding matrix $\mathbf{B}$ the derivative of the Lagrangian of (4) is equivalent to the derivative of the Lagrangian of the weighted sum MSE (WSMSE) minimization problem

$$\mathbf{B}^\star = \underset{\mathbf{B}}{\operatorname{argmin}} \quad \sum_{k=1}^{K} \operatorname{tr}\left(\mathbf{W}_k \mathbf{M}_k^{\text{MMSE}}\right) \quad \text{s.t.} \quad \operatorname{tr}(\mathbf{S}_m \mathbf{B}\mathbf{B}^H) \leq \rho_m \quad \forall m \tag{11}$$

with the fixed weighting matrices

$$\mathbf{W}_k = \nu_k \left(\mathbf{M}_k^{\text{MMSE}}\right)^{-1}. \tag{12}$$

Note, that the optimization over $\mathbf{B}$ does not affect $\mathbf{W}_k$ in (11). For a detailed derivation we refer to [12]. Based on that relation the authors presented an iterative algorithm that alternately calculates in each iteration first the MMSE receive filters (8) and the weighting matrix (12) under the given precoding matrix of the previous iteration. Afterwards the precoding matrix is optimized based on the MMSE receive filters and the weighting matrix. The algorithm is given in Table **Algorithm 1**. The algorithm of [12] uses the transmit Wiener filter approach of [15] in order to optimize the precoding matrix in step (c) of **Algorithm 1**. The filter optimization is based on the assumption that the received signals are equally scaled at the UEs ($\mathbf{U} = u\mathbf{I}$). The algorithm jointly optimizes the precoding matrix $\mathbf{B}$ and the scaling u. Instead of using $u\mathbf{I}$, MMSE receive filters (8) are applied at the UE. However, since the precoder design inherently optimizes a part of the receive filters the solution differs from the strictly independent optimization of precoding matrix and receive filters.

Algorithm 1: General approach for maximizing the WSR

```
set iteration index i = 0
initialize B^i = B^init
repeat
  update i = i + 1
  (a) update of the receive filter U_k^i | B^{i-1} ∀k
  (b) update of the weighting matrix W_k^i | B^{i-1} ∀k
  (c) update of the precoding matrix B^i | U^i, W^i
until convergence
```

A. Imperfect CSI

For the case where the channel is only imperfectly available at the CU, we are looking at the average MSE matrix obtained by substituting the channel matrix (3) within (7), which results in

$$\begin{aligned}\bar{\mathbf{M}}_k = \mathbb{E}\{\mathbf{M}_k\} &= \mathbb{E}\left\{(\mathbf{d}_k - \mathbf{U}_k\mathbf{y}_k)(\mathbf{d}_k - \mathbf{U}_k\mathbf{y}_k)^H\right\} \\ &= \mathbf{I} + \sigma_n^2 \mathbf{U}_k\mathbf{U}_k^H \\ &\quad + \mathbf{U}_k\hat{\mathbf{H}}_k\mathbf{B}\mathbf{B}^H\hat{\mathbf{H}}_k^H\mathbf{U}_k^H \\ &\quad + \mathbb{E}\left\{\mathbf{U}_k\mathbf{E}_k\mathbf{B}\mathbf{B}^H\mathbf{E}_k^H\mathbf{U}_k^H\right\} \\ &\quad - \mathbf{U}_k\hat{\mathbf{H}}_k\mathbf{B}_k - \mathbf{B}_k^H\hat{\mathbf{H}}_k^H\mathbf{U}_k^H.\end{aligned} \tag{13}$$

The expectation in (13) is taken w.r.t the error matrix $\mathbf{E}_k$, the data symbols and the noise. As stated in [14] the expected achievable user rate can be lower bounded by

$$\mathbb{E}\{-\log\det(\mathbf{M}_k)\} \geq -\log\det(\mathbb{E}\{\mathbf{M}_k\}) = \bar{R}_k. \tag{14}$$

Based on relation (14) we focus on the maximization of the average WSR lower bound (AWSR-LB)

$$\mathbf{B}^\star = \underset{\mathbf{B}}{\operatorname{argmax}} \quad \sum_{k=1}^{K} \nu_k \bar{R}_k \quad \text{s.t.} \quad \operatorname{tr}(\mathbf{S}_m\mathbf{B}\mathbf{B}^H) \leq \rho_m \quad \forall m. \tag{15}$$

The expectation over the additional error term in (13) can be eliminated with

$$\mathbb{E}\left\{\mathbf{U}_k\mathbf{E}_k\mathbf{B}\mathbf{B}^H\mathbf{E}_k^H\mathbf{U}_k^H\right\} = \mathbf{U}_k\boldsymbol{\Phi}_k\mathbf{U}_k^H \tag{16}$$

using $\boldsymbol{\Phi}_k = \operatorname{diag}^{-1}(\boldsymbol{\Sigma}_k \operatorname{diag}(\mathbf{B}\mathbf{B}^H))$ and the reshaped error covariance matrix $\boldsymbol{\Sigma}_k = [\sigma_{k,1}^2 \mathbf{1}_{U_k \times B_1}, \ldots, \sigma_{k,M}^2 \mathbf{1}_{U_k \times B_M}]$. The operator $\operatorname{diag}(\cdot)$ stacks the diagonal elements of a matrix into a column vector, while the inverse operator $\operatorname{diag}(\cdot)^{-1}$ generates a diagonal matrix out of a column vector. With transformation (16) the MMSE receive filter which minimizes $\operatorname{tr}(\bar{\mathbf{M}}_k)$ can be written as

$$\mathbf{U}_k^{\text{MMSE}} = \mathbf{B}_k^H\hat{\mathbf{H}}_k^H(\hat{\mathbf{H}}_k\mathbf{B}\mathbf{B}^H\hat{\mathbf{H}}_k^H + \boldsymbol{\Phi}_k + \sigma_n^2\mathbf{I})^{-1}. \tag{17}$$

Combining (17) with (13) results in the average MMSE matrix

$$\bar{\mathbf{M}}_k^{\text{MMSE}} = (\mathbf{I} + \mathbf{B}_k^H\hat{\mathbf{H}}_k^H\bar{\mathbf{C}}_k^{-1}\hat{\mathbf{H}}_k\mathbf{B}_k)^{-1} \tag{18}$$

with the average inter-user interference and noise covariance matrix

$$\bar{\mathbf{C}}_k = \sigma_n^2\mathbf{I} + \boldsymbol{\Phi}_k + \sum_{l=1, l\neq k}^{K} \hat{\mathbf{H}}_k\mathbf{B}_l\mathbf{B}_l^H\hat{\mathbf{H}}_k^H. \tag{19}$$

Based on the previous derivations the AWSR-LB can be maximized by adapting **Algorithm 1** with (17) in step (a), the weighting matrix $\bar{\mathbf{W}}_k = \nu_k \left(\bar{\mathbf{M}}_k^{\text{MMSE}}\right)^{-1}$ in step (b) and the robust precoder which solves the optimization problem

$$\mathbf{B}^\star = \underset{\mathbf{B}}{\operatorname{argmin}} \quad \sum_{k=1}^{K} \operatorname{tr}\left(\bar{\mathbf{W}}_k\bar{\mathbf{M}}_k\right) \quad \text{s.t.} \quad \operatorname{tr}(\mathbf{S}_m\mathbf{B}\mathbf{B}^H) \leq \rho_m \quad \forall m, \tag{20}$$

in step (c). Assuming $M = 1$, problem (20) can be solved with the transmit Wiener filter approach, which minimizes the average weighted sum MSE

$$\epsilon = \operatorname{tr}\left(\bar{\mathbf{W}}\bar{\mathbf{M}}\right), \tag{21}$$

where $\bar{\mathbf{M}} = \operatorname{blkdiag}(\bar{\mathbf{M}}_1, ..., \bar{\mathbf{M}}_K)$ is the average MSE matrix of all users and $\bar{\mathbf{W}} = \operatorname{blkdiag}(\bar{\mathbf{W}}_1, ..., \bar{\mathbf{W}}_K)$ is the overall weighting matrix. For eliminating the expectation of the error term in (13) in order to obtain the derivative of (21) w.r.t. the precoding matrix $\mathbf{B}$, we introduce the additional transformation

$$\operatorname{tr}\left(\mathbb{E}\left\{\bar{\mathbf{W}}\mathbf{U}\mathbf{E}\mathbf{B}\mathbf{B}^H\mathbf{E}^H\mathbf{U}^H\right\}\right) = \operatorname{tr}\left(\mathbf{D}\operatorname{dg}\left(\mathbf{B}\mathbf{B}^H\right)\right) \tag{22}$$

with $\mathbf{D} = \boldsymbol{\Lambda}^T \operatorname{diag}\left(\mathbf{U}^H\bar{\mathbf{W}}\mathbf{U}\right)\mathbf{1}_{1\times U}$. Matrix $\boldsymbol{\Lambda} = \mathbf{E} \odot \mathbf{E}^*$ consists of the error variances $\sigma_{k,m}^2$, $\forall k, m$ at the respective positions. With the transformation (22) the weighted robust Wiener filter that optimizes problem (20) assuming fixed

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receive filters $\mathbf{U}$ and a fixed weighting matrix $\bar{\mathbf{W}}$ results in the precoding matrix

$$\bar{\mathbf{B}} = b\mathbf{A}^{-1}\hat{\mathbf{H}}^H\mathbf{U}^H\bar{\mathbf{W}} \tag{23}$$

including an additional regularization of matrix

$$\mathbf{A} = \hat{\mathbf{H}}^H\mathbf{U}^H\bar{\mathbf{W}}\mathbf{U}\hat{\mathbf{H}} + \text{dg}\left(\mathbf{D}\right) + \frac{\sigma_n^2 \text{tr}\left(\bar{\mathbf{W}}\mathbf{U}\mathbf{U}^H\right)}{\rho}\mathbf{I}. \tag{24}$$

The precoding matrix (23) is scaled with

$$b = \sqrt{\rho/\text{tr}(\mathbf{A}^{-2}\hat{\mathbf{H}}^H\mathbf{U}^H\bar{\mathbf{W}}\bar{\mathbf{W}}^H\mathbf{U}\hat{\mathbf{H}})}, \tag{25}$$

in order to satisfy the sum power constraint. Note, that compared to [14] our solution allows different error variances for each MIMO link. Therefore, the presented single-cell precoding scheme can also be applied to multi-cell systems by consistently scaling the precoding matrix such that the transmit power constraint is satisfied at each BS.

IV. Multi-Cell Optimization

The basic difference to the previous section is that for multi-cell optimization the transmit power is restricted per BS instead of a sum power constraint. As stated in [13] the multi-cell problem can also be solved with **Algorithm 1** by adapting step (c) such that (11) can be solved for $M > 1$. Beside the solutions given in [13] a different approach for minimizing the sum MSE under per BS power constraints was presented in [16]. The solution can easily be adapted to the WSMSE minimization problem by decomposing $\mathbf{W} = \mathbf{F}\mathbf{F}^H$ and solving the following second order cone program (SOCP):

$$\begin{aligned} \mathbf{B}^\star = \quad & \underset{\forall \mathbf{B}}{\text{argmin}} \quad t \\ \text{s.t.} \quad & \left\| \begin{matrix} \text{vec}(\mathbf{F}^H\mathbf{U}\mathbf{H}\mathbf{B} - \mathbf{F}^H) \\ \sigma_n\sqrt{\text{tr}(\mathbf{W}\mathbf{U}\mathbf{U}^H)} \end{matrix} \right\|_2 \le t \\ & \|\ \text{vec}(\mathbf{S}_m\mathbf{B})\ \|_2 \le \sqrt{\rho_m} \quad \forall m. \end{aligned} \tag{26}$$

The global optimum for (26) can be obtained by using standard software solvers like SEDUMI [17].

A. Imperfect CSI

The SOCP solution presented in [16] was adapted in [10] to the case of imperfect CSI by integrating an additional regularization term into the second order cone. We adapt the solution in [10], in order to minimize the WSMSE

$$\begin{aligned} \mathbf{B}^\star = \quad & \underset{\forall \mathbf{B}}{\text{argmin}} \quad t \\ \text{s.t.} \quad & \left\| \begin{matrix} \text{vec}(\mathbf{F}^H\mathbf{U}\hat{\mathbf{H}}\mathbf{B} - \mathbf{F}^H) \\ \mathbf{\Psi}\text{vec}(\mathbf{B}) \\ \sigma_n\sqrt{\text{tr}(\mathbf{W}\mathbf{U}\mathbf{U}^H)} \end{matrix} \right\|_2 \le t \\ & \|\ \text{vec}(\mathbf{S}_m\mathbf{B})\ \|_2 \le \sqrt{\rho_m} \quad \forall m \end{aligned} \tag{27}$$

with the reshaped error matrix $\mathbf{\Psi}_k = \mathbf{I}_U \otimes \text{blkdiag}\{\mathbf{I}_{B_1} \otimes \text{vec}(\sigma_{k,1}\mathbf{F}_k^H\mathbf{U}_k), \dots, \mathbf{I}_{B_M} \otimes \text{vec}(\sigma_{k,M}\mathbf{F}_k^H\mathbf{U}_k)\}$.

V. Simulation Setup

In this section a toy scenario is investigated where $M = 2$ BSs jointly transmit to $K = 2$ UEs as illustrated in Fig. 1. Each UE is assigned to the closest BS. The distance between the BSs is d_I and the relative user position is denoted by $\delta = d/d_I$, where d is the distance between BS and its assigned UE. It is assumed that both users are placed symmetrically within the area between the two BSs. Boths BSs are allowed

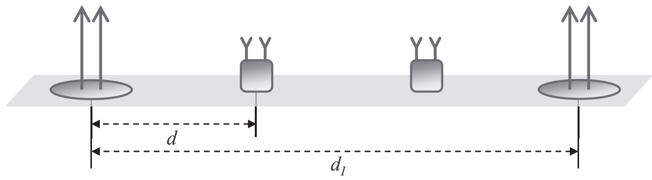

Fig. 1. Investigated toy scenario with $M = 2$ BSs and $K = 2$ UEs. The UEs are symmetrically placed within the area between the BSs

to transmit with maximum power ρ resulting in a signal-to-noise ratio (SNR) at the cell edge (CE) of

$$\text{SNR}_{\text{CE}} = \log_{10}\left(\rho\beta(d_I/2)^{-\alpha}/\sigma_n^2\right). \tag{28}$$

Further simulation parameters can be found in Table I. In Fig. 2 we plotted the average user rate $\sum_k R_k/K$ in bits per channel use over the cell edge SNR while $\delta = 0.5$. Current non-cooperative cellular systems are addressed with *reuse 1* and *reuse 2*. Both schemes do not require any CSI. Additionally, we simulated the rate performance of [12], [10] and the proposed solutions. For perfect CSI (P-CSI), our proposed single-cell (SC) solution is equivalent to [12] and performs best in the high SNR regime while our multi-cell (MC) solution performs best in the moderate and low SNR regime. That is not obvious, since the SC solution is optimized for a sum power constraint and rescaled afterwards. However, as mentioned in Sec. III the transmit Wiener filter inherently optimizes a part of the receive filter, which leads to a stronger rate gain per iteration. While in the high SNR regime the maximum number of iterations is more often achieved before the wanted accuracy is reached, in the low SNR regime less iterations are required. In the asymptotic regime in terms of number of iterations and accuracy the MC solution is expected to outperform the SC solution for the whole SNR range. The

TABLE I
Simulation Parameters

Parameter	Value
Number of BS antennas	$B_m = 2 \quad \forall m$
Number of UE antennas	$U_k = 2 \quad \forall k$
Noise power	$\sigma_n^2 = 1$
Path loss exponent	$\alpha = 3.5$
Model coefficient	$\beta = 10^{-14.5}$
Feedback + backhaul delay	$\Delta = 15$ ms
User velocity	$v = 10$ km/h
Coherence time	$T_C = 10$ ms
Inter side distance	$d_I = 500$ m
User weights	$\nu_k = 1 \quad \forall k$
Max. number of iterations	$i_{max} = 30$
Accuracy	$1 - R_{i-1}/R_i = 0.0001$

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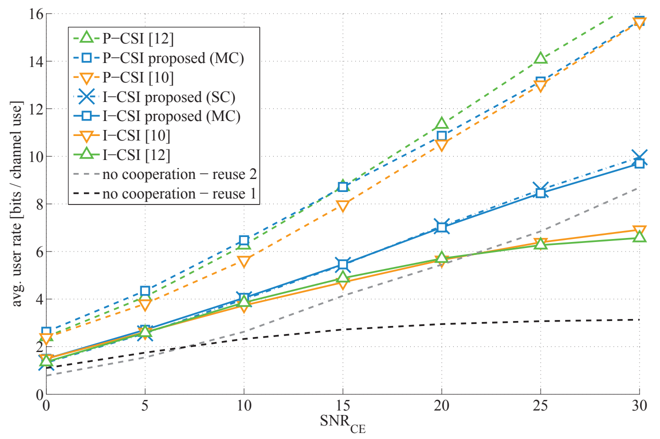

Fig. 2. Average user rate dependent on the cell edge SNR in dB, while both users are located at the cell edge ($\delta = 0.5$).

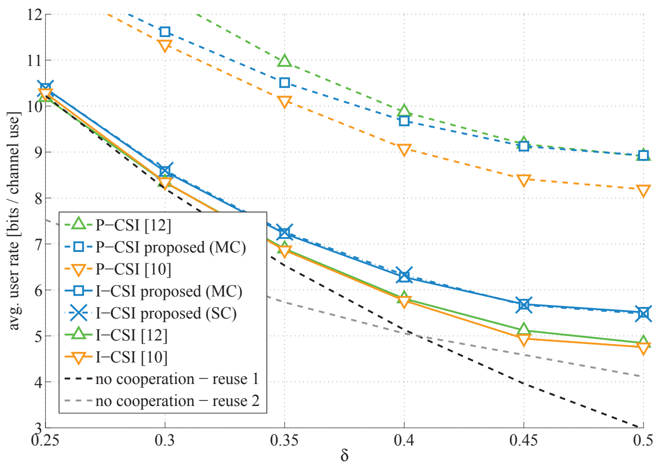

Fig. 3. Average user rate dependent on the relative UE position, while the cell edge SNR is $SNR_{CE} = 15$ dB.

algorithm of [10] performs worst in terms of rate since it is optimized in order to minimize the sum MSE. For imperfect CSI our proposed solutions clearly outperform [12] and [10] while the behavior among the SC and MC scheme can also be explained with the restriction of the iteration number. In Fig. 3 we plotted the average user rate over the relative distance δ, while the cell edge SNR is chosen to be 15 dB. Note, that the point $\delta = 0.5$ in Fig. 3 is equivalent to $\text{SNR}_{\text{CE}} = 15$ dB in Fig. 2. While our proposed schemes perform best in the cell edge area, with decreasing δ the performance of all schemes under imperfect CSI conditions converges to the non cooperative reuse 1 bound.

VI. Conclusions

In this paper we studied the linear precoding design for the multi-cell MU-MIMO downlink. We used a previous result which maximized the WSR under a sum power constraint and adapted it to per BS power constraints. We integrated a former result which gave an approach for robust WSR maximization under imperfect CSI conditions and proposed two novel precoding algorithms. Simulation results showed performance gains of our solutions compared to state of the art precoding, especially in the cell edge area.

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