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GFDM - Generalized Frequency Division Multiplexing

Invited Paper

Gerhard Fettweis, Marco Krondorf and Steffen Bittner
Vodafone Chair Mobile Communications Systems
Technische Universität Dresden
01069 Dresden, Germany
Email: {fettweis,krondorf,bittner}@ifn.et.tu-dresden.de

***Abstract*—This paper presents the GFDM system, a generalized digital multi-carrier transceiver concept. GFDM is based on traditional filter bank multi-branch multi-carrier concepts which are now implemented digitally. Our GFDM approach exhibits some attractive features which are of particular importance for scenarios exhibiting high degrees of spectrum fragmentation. Spectrum fragmentation is a typical technical challenge of digital dividend use cases, exploiting spectrum white spaces in the TV UHF bands which are located in close proximity to allocated spectrum. Specifically, the GFDM features are a lower PAPR compared to OFDM, a ultra-low out-of-band radiation due adjustable Tx-filtering and last but not least a block-based transmission using cyclic prefix insertion and efficient FFT-based equalization. GFDM enables frequency and time domain multi-user scheduling comparable to OFDM and provides an efficient alternative for white space aggregation even in heavily fragmented spectrum regions.**

I. Introduction

The presented GFDM scheme defines a transceiver architecture and a PHY concept, allowing to opportunistically exploit spectrum white spaces for wireless data communications. For instance, spectrum holes in the UHF TV bands (TVWS - TV white spaces) are a prominent scenario due to the digital dividend [1] and serves as motivating GFDM use case in this paper. The design of such opportunistic devices is particularly difficult mainly for two reasons. On the one hand, signal generation should ensure ultra-low out of band radiation to strictly avoid harmful interference to legacy TV signals. On the other hand, the receivers should exhibit high sensitivity in order to explore white spaces, i.e., to sense even very weak TV signals. Apart from these fundamental requirements, there are even more engineering challenges which have to be addressed throughout this paper.

Typically, TV white spaces are not consecutively places in the spectrum, but the UHF TV spectrum exhibits strong spectrum fragmentation. In order to efficiently exploit all detected TV white spaces, a system is required which can cope with strong spectrum fragmentation and which is able to perform aggregation of several TVWS by one single wide band signal. Hence, the GFDM transceiver is wide band and addresses the following demands:

- Low out of band radiation to avoid harmful interference to legacy TV signals
- Simple equalization despite the wideband nature of the transmit signal
- Frequency agile white space allocation, flexible signal bandwidth
- Digital implementation to reduce the requirements of the analogue front-end

Recent studies provided by the European FP6 project ORACLE [2] indicate the usage of multi-carrier systems to flexibly exploit vacant spectrum by switching subcarriers on and off. However, the most prominent multi-carrier system OFDM is known to cause strong spectral leakage even when using pulse shaping techniques or guard carriers. Hence GFDM aims at combining the flexibility and simplicity of OFDM with stronger interference reduction mechanisms.

II. GFDM Transmitter

GFDM is a multi-carrier system, which digitally implements the classical filter band approach. Cyclic prefix (CP) insertion is used to allow for low complex equalization at the receiver side. According to Fig.1, a tail biting technique is used to shorten the cyclic prefix in order to enhance the spectral efficiency. Every subcarrier is modulated individually, using some form of QAM signalling. Let's denote the QAM symbol stream on

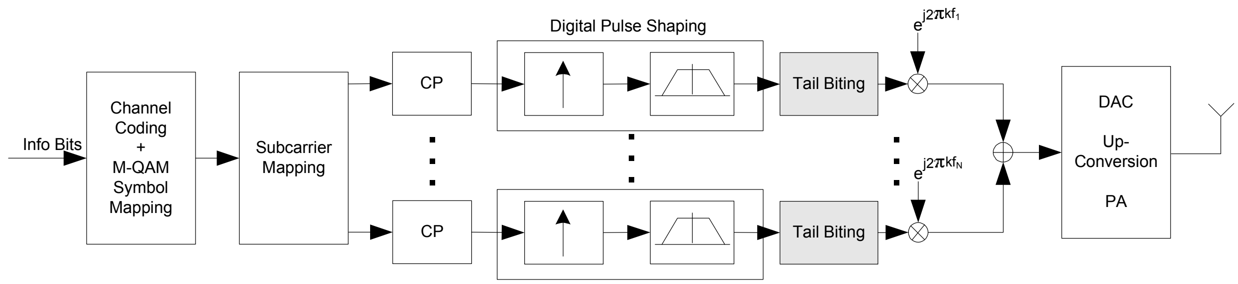

Fig. 1. Principle of the GFDM digital transmitter

subcarrier n as

$$s(n,k) \ ,$$

where k represents the QAM symbol index. After up-sampling, the symbols index now turns into the sample index k', representing the sample duration T_S.
Subsequently, cyclic prefix insertion is performed, accounting for the filter length of the digital pulse shaping, the filter length of the digital receive filter and the length of the mobile channel impulse response. After cyclic prefix insertion, digital pulse shaping is performed subcarrier-wise as follows

$$s(n,k') * g_{Tx}(n,k') \ , \qquad (1)$$

which is crucial to yield low out of band radiation. Here,

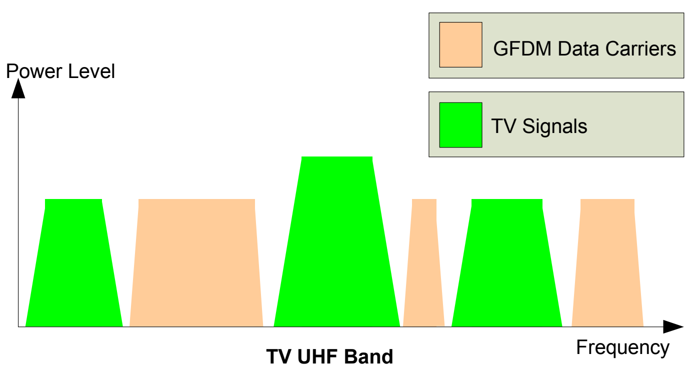

Fig. 2. Principle of opportunistic TVWS allocation

$*$ denotes the convolution operation with respect to k'. Sharp filter edges are required which in turn necessitate high Tx-filter orders. Large filter orders are generally problematic due to the cyclic prefix, which has to be matched to the aggregate filter lengths of all system filters involved. However, for the digital Tx-filter, a tail-biting technique according to [3] can be applied in order to reduce the CP overhead as depicted in Fig.3. Although not studied in this paper, a CP reduction for the Rx filter can be done as well, if pre-coding techniques are applied. One drawback of such a strategy is a potentially increasing transmit power, which can be mitigated for example when using non-linear pre-coding techniques and modulo constellations as presented in [4] and [5].

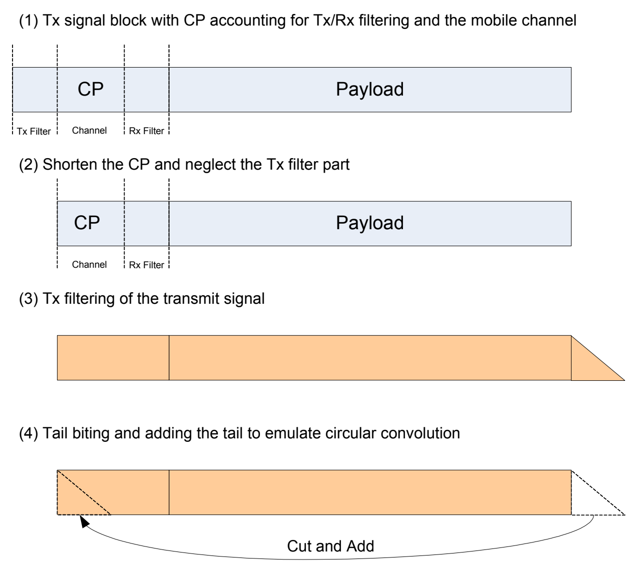

Fig. 3. Principle of CP shortening by tail biting

After individual subcarrier pulse shaping, each carrier n is digitally shifted to its carrier frequency f_n, which is normalized to the signal bandwidth $B = 1/T_S$ and hence is defined in the interval $[-1/2 : 1/2)$. The resulting time domain signal $x(k')$ hence becomes

$$x(k') = \sum_n (s(n,k') * g_{Tx}(n,k'))\mathrm{e}^{j2\pi k' f_n} \ . \qquad (2)$$

Finally, $x(k')$ is digital-to-analogue converted, mixed to the carrier frequency, amplified and transmitted.
It is well known that OFDM uses a cyclic prefix to simplify signal equalization but exhibits a high PAPR as one major drawback. A lower PAPR and simple equalization is achieved when using a cyclic prefix in conventional single carrier systems, leading to the SC-CP (single carrier with cyclic prefix) system concept [6] as applied in LTE uplinks [7]. Hence, from the above

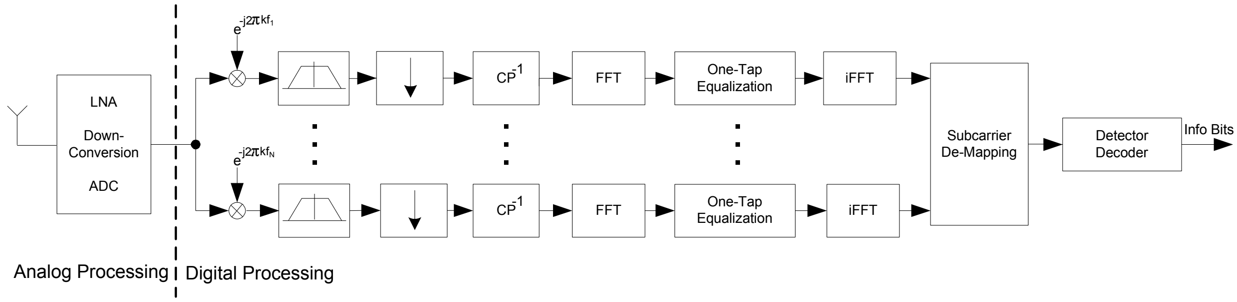

Fig. 4. Principle of the GFDM digital receiver

given GFDM signal generation scheme we can interpret GFDM as parallel SC-CP system realized in the digital domain. There are numerous consequences from this observation:

- Each subcarrier represents an independent SC-CP link, which can be modulated individually having its own bandwidth and pulse shaping.
- The GFDM carriers are no longer orthogonal but exhibit mutual interference which can be adjusted by the individual Tx-filters and signal bandwidths.
- Due to the digital realization, GFDM combines the simple equalization of OFDM and SC-FC with the ability of flexible white space allocation and controllable out of band radiation.

III. GFDM Receiver

The GFDM receiver as depicted in Fig.4 performs parallel SC-CP demodulation for each of the GFDM subcarriers. After passing the LNA and the down conversion stage, the received signal is analogue-to-digital converted from which we obtain the digitized received signal $y(k')$. Here, $y(k')$ is given by the discrete convolution of the Tx-signal $x(k')$ with the channel impulse response $h(k')$, corrupted by zero-mean additive white Gaussian noise $n(k')$ with variance σ_n^2:

$$y(k') = x(k') * h(k') + n(k'), \tag{3}$$

where we define the SNR γ as follows:

$$\gamma = \frac{\mathrm{E}\{|x(k') * h(k')|^2\}}{\sigma_n^2} .$$

The received signal is then mixed to baseband individually in each of the digital receiver branches and filtered subsequently, yielding the signal $z(n, k')$:

$$z(n, k') = \left(y(k')\mathrm{e}^{-j2\pi k' f_n}\right) * g_{Rx}(n, k') . \tag{4}$$

Here the Rx-filtering is used to cancel out the undesired adjacent channel interference. A high selectivity, i.e., a steep filter edges, will provide low inter channel interference (ICI) and therefore higher SNR. However, sharp filter edges typically require high filter orders, which in turn have to be compensated by an appropriate cyclic prefix length, decreasing the system spectral efficiency. After digital Rx-filtering, the signal is down-sampled turning the sample index k' back into the QAM symbol index k. The cyclic prefix is removed and the signal $z(n, k')$ is transformed using FFT in order to yield the frequency bins $Z(n, l)$ of the n-th subcarrier signal. The signal model can now be written as:

$$Z(n, l) = S(n, l)H(n, l) + W(n, l) , \tag{5}$$

where $S(n, l)$ is the FFT transformed version of the data signal $s(n, k)$ and $W(n, l)$ is the l-th frequency bin of the Rx-filtered noise and ICI. The channel transfer function $H(n, l)$ denotes the FFT transformed effective channel, consisting of the Tx-filter, the Rx-filter and the mobile channel:

$$H(n, l) = \mathrm{FFT}_l \left\{g_{Tx}(n, k) * h(k) * g_{Rx}(n, k)\right\} . \tag{6}$$

Equalization can now be achieved e.g. by zero-forcing operation

$$\hat{S}(n, l) = Z(n, l)/H(n, l), \tag{7}$$

where the equalized data signal $\hat{s}(n, k)$ is obtained from IFFT, which is then fed in the detector/decoder stage.

IV. GFDM Performance Evaluation

In this section, we compare the GFDM bit error rate (BER) performance with the corresponding OFDM BER. The aim of this evaluation is to show the impact of the non-orthogonal GFDM carriers which can be controlled by the selectivity of the Tx and Rx digital system filters. The channel transfer function $\mathrm{FFT}\left\{g_{Tx}(n, k) * h(k) * g_{Rx}(n, k)\right\}$ is assumed to be perfectly known at the receiver where we consider an AGWN environment. This means that the channel impulse response $h(k)$ only consists of one tap which is always set to $h(0) = 1$. Both the OFDM and the GFDM system are designed to have the same spectral efficiency, i.e., both system parameter sets are defined according to Table I.

In our numerical example, we use a CP overhead of 25%

Parameter	OFDM	GFDM
Signal Bandwidth	20 MHz	20 MHz
Number of Subcarriers	64	4
Subcarrier bandwidth	312.5 kHz	5 MHz
Symbol duration w/o CP	3.2 μs	0.2 μs
CP overhead	25%	25%
CP length	16 Samples	Variable
Pulse Shape	rectangular	FIR: Cos-roll r=0.1

TABLE I
SYSTEM PARAMETER SETS FOR GFDM AND OFDM

for both GFDM and OFDM. OFDM inherently exhibits this overhead due to the CP of each OFDM symbol. Conversely, in the GFDM system the CP depends on the length of the Rx-filter, where the length of the mobile channel can be neglected due to the AWGN assumption. Assuming a CP overhead of 25% and a Rx-filter order of 25 symbols, the GFDM FFT block length is 100 symbols. Hence, the GFDM FFT is 6.25 times longer than in the OFDM system.

Fig.5 compares the uncoded QPSK BER for both OFDM and GFDM, where GFDM uses different filter orders for Tx and Rx filtering. The orders of the digital Tx and Rx cosine roll-off filters are denoted by O_{Tx} and O_{Rx} respectively. Clearly, the GFDM BER performance is slightly worse due to the orthogonality loss, which can be controlled by appropriate filter designs.

Fig.6 compares the peak-to-average power ratio (PAPR)

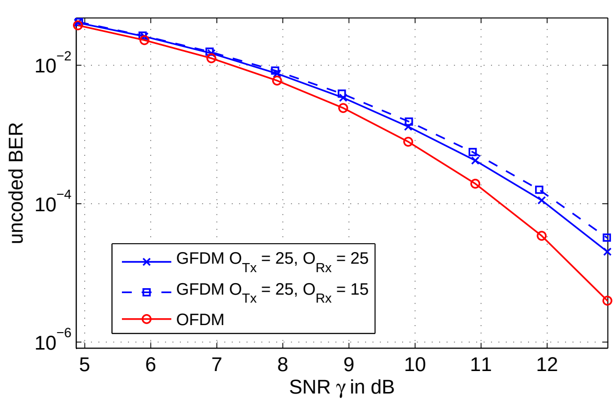

Fig. 5. Bit error rate performance for QPSK modulation, AWGN channel used

of OFDM and GFDM. Due to less subcarriers, GFDM exhibits a superior PAPR performance compared to conventional OFDM signalling.

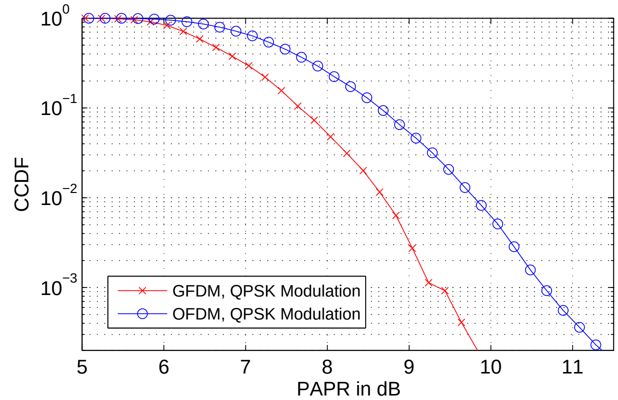

Fig. 6. CCDF of PAPR for GFDM and OFDM

V. CONCLUSION

In this paper we presented a multi-carrier system architecture based on digitally implemented filter banks. It is shown that GFDM significantly reduces the requirements set on the analogue front end. In particular GFDM combines both, the advantage of specific subcarrier allocation and low PAPR. Low PAPR allows to reduce the hardware cost and power consumption, which is an important point of sale for future wireless systems. Furthermore, each single subcarrier can be modulated individually, which provides a high degree of flexibility in the system design and allows for efficient multi-user scheduling. Compared to conventional multi-carrier system, we show that GFDM is a promising alternative for heavily fragmented spectrum.

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