Outage Based Power Allocation for a Lossy Forwarding Two-Relaying System

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***Abstract*—The extension of Decode-and-Forward (DF) relaying by lossy forwarding has the potential to ensure a reliable multi-hop message transport in wireless mesh networks. Unlike in conventional DF relaying, with lossy forwarding a relay forwards a message regardless whether errors have been detected after decoding. At the destination, a proper joint decoding technique exploits the high correlation of messages received via different network paths. According to the Slepian-Wolf correlated source coding theorem a performance improvement compared with the conventional DF relaying can be expected. The performance can be optimized by a power allocation scheme that distributes the total transmit power budget between source and relay nodes. This paper analyzes the outage probability (OP) based on the Slepian-Wolf source correlation theorem for a system with two relays and designs a power allocation scheme to minimize the OP. The proposed scheme reduces the OP by up to 1.5 orders of magnitude compared to the reference case of equal power allocation. We also compare the performance gain of a system with two relays against the case with a single relay for the same total transmit power budget. Results show a reduction of the OP of at least one and up to two orders of magnitude.**

I. Introduction

Wireless networks are confronted with enormous challenges when the communication infrastructure of mobile cellular networks fails. For example, in events of environmental disasters the communication infrastructure of mobile cellular networks can be damaged, which may lead to a collapse of the communication system. In this scenario, intact mobile devices can establish a mesh network without the need for central coordination and a backbone infrastructure. The system poses high requirements on the energy efficiency and reliable information transfer, i.e. mobile devices are confined by the limited energy resources. Therefore, the transmit power should be reduced to ensure long participation of all mobile devices in the network. Evidently, a low signal-to-noise ratio (SNR) occurs and results in unreliable information transfer.

Cooperative communication techniques emerge as promising strategies to ensure reliable data transport in mesh networks. They utilize spatial diversity by sending data via intermediary relay nodes between source and destination, which forward the data [1]. Nevertheless, conventional Decode-and-Forward (DF) schemes are suboptimal, since the relays verify that the received data is not corrupted and eventually discard erroneous messages. However, lossy links in mesh networks are inevitable, and energy consuming retransmissions are needed to ensure reliable multi-hop connectivity.

In order to improve energy efficiency and establish a reliable information transfer in mesh networks, an innovative distributed source coding (DSC) scheme has been proposed in [2]. The DSC scheme exploits that an erroneous message at the relay is highly correlated with the original message and can consequently function as a helper in the decoding process at the destination. The performance gain can be reasoned with the Slepian-Wolf correlated source coding theorem [3]. In this study, the link between source and relay is considered to be lossy and characterized by a bit flipping probability p_i [4]. At the relay the erroneous message is re-encoded, interleaved and forwarded to the destination. At the destination the joint decoding technique presented in [2] exploits the correlation between original message and erroneous message with a likelihood ratio update function. A significant improvement of the decoding performance can be observed [2].

The outage probability (OP) for a *lossy forwarding one-relaying* (LFOR) system based on the Slepian-Wolf correlated source coding theorem with block Rayleigh fading (BRF) channel has been analyzed in [5]. More recently, a power allocation strategy to minimize the OP has been proposed in [6]. In the present paper, we extend the LFOR system considered in [5] to a *lossy forwarding two-relaying* (LFTR) system and derive the OP based on the Slepian-Wolf correlated source coding theorem. Similar to [6] we propose a power allocation strategy to minimize the OP for the LFTR system.

The remainder of the paper is organized as follows: Sec. II describes the LFTR system model and introduces the Slepian-Wolf admissible rate region for the LFTR system. Sec. III derives the OP. Sec. IV introduces the power allocation strategy based on convex optimization. Sec. V verifies the OP derivation and power allocation strategy by means of a Monte-Carlo simulation and compares the LFTR system to the LFOR system. Sec. VI summarizes the results of the paper.

II. System Model

A. LFTR System Model

We consider a half-duplex relay system, where the source (S) and two relays (R) cooperate to transmit to the destination (D) as shown in Fig. 1. To ensure orthogonal transmission, time division multiple access (TDMA) is assumed. The source encodes and broadcasts an information sequence $\mathbf{b}_0$ with sequence length N to both relays and the destination. The information sequence is an independent and identically distributed (i.i.d.) sequence with $\Pr[\mathbf{b}_0[n] = 0] = \Pr[\mathbf{b}_0[n] = 1] = 0.5$. Each relay decodes the received information sequence. The decoded information sequences $\mathbf{b}_1$ and $\mathbf{b}_2$ can differ from the source information sequence depending on the channel states between source and relays. Nevertheless, $\mathbf{b}_1$ and $\mathbf{b}_2$ are highly correlated with $\mathbf{b}_0$ [7]. The information sequences at the relays are interleaved, re-encoded and forwarded to the destination. Allowing the transmission of

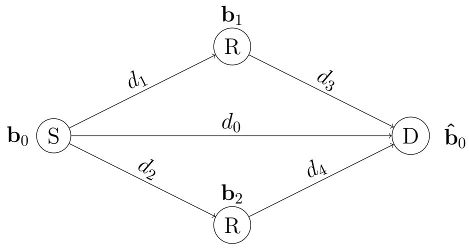


Fig. 1: Lossy forwarding two-relaying system model.

erroneous information sequences at the relay is also referred to as *lossy forwarding* [5]. All received information sequences $\mathbf{b}_0$, $\mathbf{b}_1$ and $\mathbf{b}_2$ are jointly decoded at the destination to retrieve $\mathbf{b}_0$. The joint decoder can exploit the correlation of the information sequences and achieve tremendous performance gain in terms of the estimated source information sequence $\hat{\mathbf{b}}_0$ [2].

All links are assumed to be affected by independent BRF and additive white Gaussian noise (AWGN) with mean power N_0. The probability density function (pdf) of the instantaneous SNR γ_i is

$$p(\gamma_i) = \frac{1}{\Gamma_i}\exp(-\frac{\gamma_i}{\Gamma_i}), i \in \{0, ..., 4\} \tag{1}$$

with average SNR between source and relays, source and destination

$$\Gamma_i = \frac{E_0}{N_0} \cdot G_i, i \in \{1, 2, 3\} \tag{2}$$

and average SNR between relays and destination

$$\Gamma_j = \frac{E_i}{N_0} \cdot G_j, (i, j) \in \{(1, 3), (2, 4)\}, \tag{3}$$

where E_i is the transmit power of source ($i = 0$) or relay i ($i = 1, 2$), respectively. The geometrical gain $G_i = ({}^{d_i}/_{d_0})^{-\eta}$ depends on the distances d_i between source, relays and destination as shown in Fig. 1. The geometrical gain is normalized to the distance between source and destination d_0. The path loss exponent η is 3.52 to represent urban and suburban areas, determined empirically in [8].

The bit flipping probability $p_i = \Pr[\mathbf{b}_0[n] \neq \mathbf{b}_i[n]]$, $i \in \{1, 2\}$ defines the degree of correlation between source and relay information sequences. According to Shannon's lossy source-channel separation theorem and Hamming's distortion measure [5] the bit flipping probability can be determined with the instantaneous SNR γ_i of the BRF channel

$$p_i(\gamma_i) = \begin{cases} H_b^{-1}\left(1 - \Phi(\gamma_i)\right), & \text{for } \Phi^{-1}(0) \leq \gamma_i \leq \Phi^{-1}(1) \\ 0, & \text{for } \gamma_i \geq \Phi^{-1}(1) \end{cases} \tag{4}$$

where $\Phi(\gamma_i) = \frac{1}{R_c}\log_2(1 + \gamma_i)$, and $\Phi^{-1}(\cdot)$ is the inverse function of $\Phi(\cdot)$. R_c represents the spectrum efficiency, including channel coding rate and modulation multiplicity. R_c is assumed to be the same for all links. $H_b^{-1}(\cdot)$ is the inverse function of the binary entropy function $H_b(x) = -x\log_2(x) - (1-x)\log_2(1-x)$.

B. Slepian-Wolf Admissible Rate Region

The joint decoder performance gain presented in [2] is based on the Slepian-Wolf correlated source coding theorem [3]. The source and all relays are considered to be correlated sources with transmission rates R_i, respectively. According to the Slepian-Wolf theorem, $\mathbf{b}_0$ can be recovered with arbitrary small error rate, if the transmission rates satisfy the inequality constraints [9]

$$\begin{aligned} R_0 \geq & H_b(\mathbf{b}_0 \mid \mathbf{b}_1, \mathbf{b}_2) \\ = & H_b(p_1(\gamma_1)) + H_b(p_2(\gamma_2)) \\ & - H_b(q_{1,2}(\gamma_1, \gamma_2)), \end{aligned} \tag{5}$$

$$\begin{aligned} R_0 + R_1 \geq & H_b(\mathbf{b}_0, \mathbf{b}_1 \mid \mathbf{b}_2) \\ = & H_b(p_1(\gamma_1)) + H_b(p_2(\gamma_2)), \end{aligned} \tag{6}$$

$$\begin{aligned} R_0 + R_2 \geq & H_b(\mathbf{b}_0, \mathbf{b}_2 \mid \mathbf{b}_1) \\ = & H_b(p_1(\gamma_1)) + H_b(p_2(\gamma_2)), \end{aligned} \tag{7}$$

$$\begin{aligned} R_0 + R_1 + R_2 \geq & H_b(\mathbf{b}_0, \mathbf{b}_1, \mathbf{b}_2) \\ = & 1 + H_b(p_1(\gamma_1)) + H_b(p_2(\gamma_2)), \end{aligned} \tag{8}$$

or R_0 satisfies the inequality constraint

$$R_0 \geq H_b(\mathbf{b}_0) = 1, \tag{9}$$

with cross-over probability $q_{1,2}(\gamma_1, \gamma_2) = p_1(\gamma_2) + p_2(\gamma_2) - 2p_1(\gamma_1)p_2(\gamma_2)$ [10]. The binary entropy of a vector is defined as $H_b(\mathbf{b}) = {}^{H(\mathbf{b})}/_{N}$. All transmission rates which satisfy (5) - (8) or (9) are referred to as the Slepian-Wolf admissible rate region. Assuming channel codes with Gaussian codebooks, the relationship between rate R_i and instantaneous SNR is given by [4]

$$R_i = \Phi(\gamma_j), (i, j) \in \{(0, 0), (1, 3), (2, 4)\}. \tag{10}$$

Finally, the system model presented in Fig. 1 can be expressed comprehensively by the instantaneous and average SNRs of the BRF channels.

III. Derivation of the Outage Probability

A. LFTR system

The OP of the LFTR system is defined by the Slepian-Wolf inadmissible rate region which includes all tupels of (R_0, R_1, R_2) that violate at least one inequality constraint in (5) - (8) or (9). The overall OP consists of nine OPs

$$P_{2,out} = \sum_{n=1}^{9} P_n, \tag{11}$$

where the index 2 indicates two deployed relays. These OPs are determined by the joint pdf, which can be simplified by the product of the individual pdfs of the independent BRF channels $p(\gamma_0, ..., \gamma_4) = \prod_{i=0}^{4} p(\gamma_i)$. All OPs can be calculated with a five dimensional integral

$$P_n = \int\limits_{\gamma_0} \cdots \int\limits_{\gamma_4} \prod_{i=0}^{4} p(\gamma_i) d\gamma_4 \cdots d\gamma_0, \tag{12}$$

where the integral boundaries determine the OP value. Due to the property of (4), four cases are defined by different regions of the bit flipping probabilities (cf. Tab. I). For the cases 1,2

TABLE I: Case distinction

Case	Bit Flipping Probability	Figure
1	$p_1 = 0, p_2 = 0$	Fig. 2a
2	$p_1 = 0, 0 < p_2 \leq 0.5$	Fig. 2b
3	$0 < p_1 \leq 0.5, p_2 = 0$	Fig. 2b
4	$0 < p_1 \leq 0.5, 0 < p_2 \leq 0.5$	Fig. 2c

TABLE II: Reduced entropies based on case distinction

Case	1	2	3	4
$H(\mathbf{b}_0 \mid \mathbf{b}_1, \mathbf{b}_2)$	0	0	0	$H_b(p_1) + H_b(p_2)$ $-H_b(q_{1,2})$
$H(\mathbf{b}_0, \mathbf{b}_1 \mid \mathbf{b}_2)$	0	$H_b(p_2)$	0	$H_b(p_1) + H_b(p_2)$
$H(\mathbf{b}_0, \mathbf{b}_2 \mid \mathbf{b}_1)$	0	0	$H_b(p_1)$	$H_b(p_1) + H_b(p_2)$
$H(\mathbf{b}_0, \mathbf{b}_1, \mathbf{b}_2)$	1	$1 + H_b(p_2)$	$1 + H_b(p_1)$	$1 + H_b(p_1) + H_b(p_2)$

and 3 the entropies defined in (5) - (8) can be reduced. The entropies for case 4 can not be simplified, since both relays observe the source information sequence erroneously. Tab. II summarizes the entropies for all cases.

In the following the spectrum efficiency R_c is assumed to be 1, corresponding to channel rate $1/2$ and quadrature phase-shift keying (QPSK).

Case 1 | Both relays observe the source information sequence error-free. The OP P_1 is presented in Fig. 2a. The OP boundaries for the channels shown in Fig. 2a can be converted to the instantaneous SNR integral boundaries

$$\begin{aligned} P_1 = \Pr[&0 < R_0 < 1, 0 < R_1 < 1 - R_0, \\ &0 < R_2 < 1 - R_0 - R_1, p_1 = 0, p_2 = 0] \\ = \Pr[&0 < \gamma_0 < 1, 0 < \gamma_3 < 2^{1-\Phi(\gamma_0)} - 1, \\ &0 < \gamma_4 < 2^{1-\Phi(\gamma_0)-\Phi(\gamma_3)} - 1, \\ &1 \leq \gamma_1 < \infty, 1 \leq \gamma_2 < \infty]. \end{aligned} \tag{13}$$

Case 2, Case 3 | One relay observes the source information sequence error free while the other relay observes an erroneous version of the information sequence. Fig. 2b presents the OPs P_2 and P_3 for case 2. Case 3 is the symmetric complement to case 2 and the OPs P_4 and P_5 can be calculated accordingly.

For the OPs P_2 and P_3 the channel boundaries are presented in Fig. 2b and can be converted to the instantaneous SNR integral boundaries

$$\begin{aligned} P_2 = \Pr[&0 < R_0 < H_b(p_2), 0 < R_1 < H_b(p_2) - R_0, \\ &0 < R_2 < \infty, p_1 = 0, 0 < p_2 \leq 0.5] \\ = \Pr[&0 < \gamma_0 < 2^{1-\Phi(\gamma_2)} - 1, \\ &0 < \gamma_3 < 2^{1-\Phi(\gamma_0)-\Phi(\gamma_2)} - 1, \\ &0 < \gamma_4 < \infty, 1 \leq \gamma_1 < \infty, 0 \leq \gamma_2 < 1], \end{aligned} \tag{14}$$

$$\begin{aligned} P_3 = \Pr[&0 < R_0 < 1, \max(H_b(p_2) - R_0, 0) < R_1 < \\ &1 + H_b(p_2) - R_0, 0 < R_2 < 1 + H_b(p_2) - R_0 - R_1, \\ &p_1 = 0, 0 < p_2 \leq 0.5] \\ P_3 = \Pr[&0 < \gamma_0 < 2^{1-\Phi(\gamma_2)} - 1, \\ &\max(2^{1-\Phi(\gamma_0)-\Phi(\gamma_2)} - 1, 0) < \gamma_3 < \\ &2^{2-\Phi(\gamma_0)-\Phi(\gamma_2)} - 1, \\ &0 < \gamma_4 < 2^{2-\Phi(\gamma_0)-\Phi(\gamma_3)-\Phi(\gamma_2)} - 1, \\ &1 \leq \gamma_1 < \infty, 0 \leq \gamma_2 < 1]. \end{aligned} \tag{15}$$

Case 4 | Both relays observe an erroneous version of the source information sequence. The binary entropy of the cross-over probability $H_b(q_{1,2}) = H_b(p_1 + p_2 - 2p_1p_2)$ depends on p_1 and p_2. Consequently, the integral boundaries contain the inverse binary entropy function $H_b^{-1}(\cdot)$ and $\Phi(\cdot)$ which is a logarithmic function. These non-linearities make the exact analytical integration difficult to achieve. Therefore an approximation of the binary entropy is introduced

$$\begin{aligned} H_b(q_{1,2}) &= H_b(p_1 + p_2 - 2p_1p_2) \\ &\approx \max\left(H_b(p_1), H(p_2)\right) \end{aligned} \tag{16}$$

with the relative approximation error

$$\epsilon(p_1, p_2) = \frac{\mid H_b(q_{1,2}) - \max\left(H_b(p_1), H(p_2)\right) \mid}{H_b(q_{1,2})}. \tag{17}$$

The relative approximation error is below 10 % if $p_1 \gtrless p_2 \pm \delta$, with $\delta = 0.1$ and thereby negligible for the error propagation. However, the approximation contingents an error if $p_2 - \delta < p_1 < p_2 + \delta$ and therefore the relative approximation error propagation is evaluated in the Sec. V.

Fig. 2c presents the OPs P_6, P_7, P_8 and P_9. With the approximation of (16) the integral boundaries are simplified to

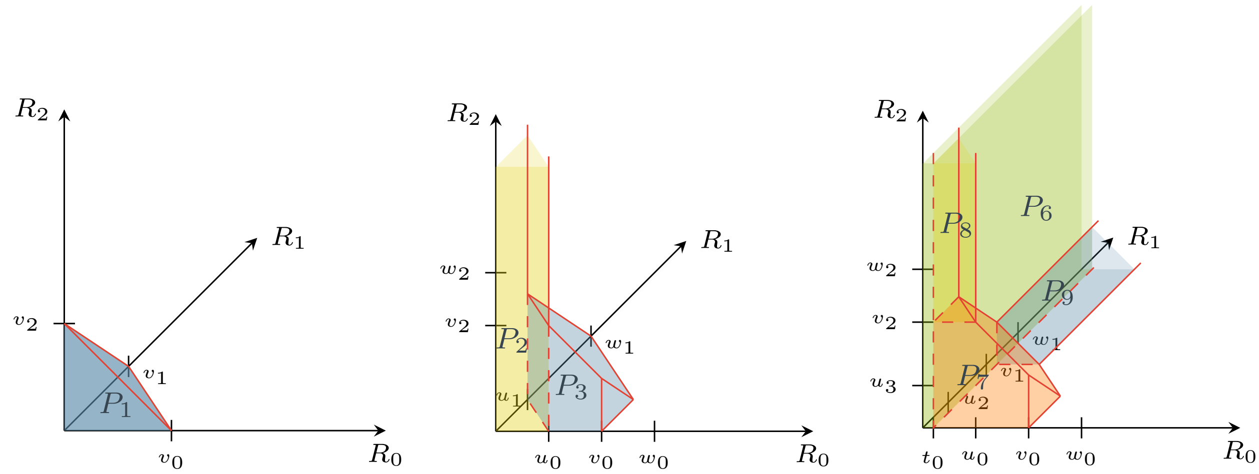
$$\begin{aligned} P_6 = \Pr[&0 < R_0 < \min(H_b(p_1), H_b(p_2)), 0 < R_1 < \infty, \\ &0 < R_2 < \infty, 0 < p_1 \leq 0.5, 0 < p_2 \leq 0.5] \\ = \Pr[&0 < \gamma_0 < 2^{\min(1-\Phi(\gamma_1), 1-\Phi(\gamma_2))} - 1, 0 < \gamma_3 < \infty, \\ &0 < \gamma_4 < \infty, 0 \leq \gamma_1 < 1, 0 \leq \gamma_2 < 1], \end{aligned} \tag{18}$$

$$\begin{aligned} P_7 = \Pr[&\min(H_b(p_1), H_b(p_2)) < R_0 < 1, \\ &0 < R_1 < \min(1 + H_b(p_1) + H_b(p_1) - R_0, 1), \\ &0 < R_2 < \min(1 + H_b(p_1) + H_b(p_1) - R_0 - R_1, 1), \\ &0 < p_1 \leq 0.5, 0 < p_2 \leq 0.5] \\ = \Pr[&2^{\min(1-\Phi(\gamma_1), 1-\Phi(\gamma_2))} - 1 < R_0 < 1, \\ &0 < \gamma_3 < \min(2^{2-\Phi(\gamma_0)-\Phi(\gamma_1)-\Phi(\gamma_2)} - 1, 1) \\ &0 < \gamma_4 < \min(2^{2-\Phi(\gamma_0)-\Phi(\gamma_3)-\Phi(\gamma_1)-\Phi(\gamma_2)} - 1, 1) \\ &0 \leq \gamma_1 < 1, 0 \leq \gamma_2 < 1], \end{aligned} \tag{19}$$

$$\begin{aligned} P_8 = \Pr[&\min(H_b(p_1) + H_b(p_2)) < R_0 < H_b(p_1) + H_b(p_2), \\ &0 < R_1 < H_b(p_1) + H_b(p_2) - R_0, 1 < R_2 < \infty, \\ &0 < p_1 \leq 0.5, 0 < p_2 \leq 0.5] \\ = \Pr[&2^{\min(1-\Phi(\gamma_1), 1-\Phi(\gamma_2))} - 1 < R_0 \\ &< 2^{2-\Phi(\gamma_1)-\Phi(\gamma_2)} - 1, \\ &0 < \gamma_3 < 2^{2-\Phi(\gamma_0)-\Phi(\gamma_1)-\Phi(\gamma_2)} - 1 \\ &1 < \gamma_4 < \infty, 0 \leq \gamma_1 < 1, 0 \leq \gamma_2 < 1]. \end{aligned} \tag{20}$$

P_9 is similar to P_8 and can be calculated by interchanging relay 1 and relay 2.

Unfortunately, the exact close-form expressions for the



(a) (b) (c)

Fig. 2: Slepian-Wolf inadmissible rate region for case 1 (a), where $v_0 = 1$, $v_1 = 1$, $v_2 = 1$, case 2 (b), where $u_0 = H_b(p_2)$, $u_1 = H_b(p_2)$, $v_0 = 1$ $w_0 = 1 + H_b(p_2)$, $v_2 = 1$, $w_1 = 1 + H_b(p_2)$, $w_2 = 1 + H_b(p_2)$ and case 4 (c), where $t_0 = H_b(p_1) + H_b(p_2) - H_b(q_{1,2}) \approx \min(H_b(p_1), H_b(p_2))$, $u_0 = H_b(p_1) + H_b(p_2)$, $u_1 = H_b(q_{1,2})$, $u_2 = H_b(q_{1,2})$, $v_0 = 1$, $v_1 = 1$, $v_2 = 1$, $w_0 = 1 + H_b(p_1) + H_b(p_2)$, $w_1 = 1 + H_b(p_1) + H_b(p_2)$, $w_2 = 1 + H_b(p_1) + H_b(p_2)$.

OPs are not easily achievable. To proceed, we characterize the high-SNR behaviour. We rewrite the OP equations by using the MacLaurin series of the exponential function $\exp(-x) \approx 1 - x$ for $x \ll 1$ [11, Eq. (1.211.1)] and only consider the lowest-order terms. The OP reduces to

$$P_{2,out} \approx \frac{C_1}{\Gamma_0\Gamma_3\Gamma_4} + \frac{1}{\Gamma_2}\frac{C_1}{\Gamma_0\Gamma_3} + \frac{1}{\Gamma_1}\frac{C_1}{\Gamma_0\Gamma_4} + \frac{1}{\Gamma_1\Gamma_2}\frac{C_2}{\Gamma_0}, \quad (21)$$

where $C_1 = 1 + \ln(2)^2 - 2\ln(2)$ and $C_2 = 3 - 4\ln(2)$. For reasons of interpretation we define two probability types, which are contained in every term of the sum in (21). The case probability $P_{\mathrm{C},i}$, defined by the regions of the bit flipping probability and the rate probability $P_{\mathrm{R},i}$, defined by the region probability of the rates R_0, R_1 and R_2. The probability that both relays observe the source error-free (case 1) has an approximate probability of $P_{\mathrm{C},1} = \int_{\gamma_1=1}^{\infty}\int_{\gamma_2=1}^{\infty} p(\gamma_1)p(\gamma_2)d\gamma_2 d\gamma_1 \approx 1$ (cf. Tab. III). However the rate probability, that at least one inequality constraint (5) - (8) is violated by the tuple (R_0, R_1, R_2) in case 1 is rather small, i.e. $P_{\mathrm{R},1} = {}^{C_1}/_{\Gamma_0\Gamma_3\Gamma_4}$ (cf. P_1 in Fig. 2a). The probability of case 2 is $P_{\mathrm{C},2} \approx \frac{1}{\Gamma_2}$ which is significant lower with respect to $P_{\mathrm{C},1}$. However, the rate probability $P_{\mathrm{R},2} = {}^{C_1}/_{\Gamma_0\Gamma_3}$ is significantly higher with respect to $P_{\mathrm{R},1}$. P_2 is the important OP for case 2, P_3 can be neglected. The same applies to case 3, where P_4 defines the OP. Case 4 is even less likely than case 2, $P_{\mathrm{C},4} = {}^{1}/_{\Gamma_1\Gamma_2}$. However, the rate probability $P_{\mathrm{R},2} = {}^{C_2}/_{\Gamma_0}$ is high and determined by OP P_6 in Fig. 2c. Eventually, case 1 is most likely to occur, however it is unlikely that an outage occurs in this case and vice versa for case 4. Case 2 and case 3 lie in between. Tab. III presents the approximated OP in (21) split into case and rate probabilities.

B. LFOR system

The exact OP for a LFOR system is derived in [5] and can be approximated for the high-SNR behaviour [12] to

TABLE III: Outage probability split into case and rate probabilities

Case i	Case probability $P_{\mathrm{C},i}$	Rate probability $P_{\mathrm{R},i}$
1	$\left(1 - \frac{1}{\Gamma_1}\right)\left(1 - \frac{1}{\Gamma_2}\right) \approx 1$	$\frac{C_1}{\Gamma_0\Gamma_3\Gamma_4}$
2	$\left(1 - \frac{1}{\Gamma_1}\right)\frac{1}{\Gamma_2} \approx \frac{1}{\Gamma_2}$	$\frac{C_1}{\Gamma_0\Gamma_3}$
3	$\frac{1}{\Gamma_1}\left(1 - \frac{1}{\Gamma_2}\right) \approx \frac{1}{\Gamma_1}$	$\frac{C_1}{\Gamma_0\Gamma_4}$
4	$\frac{1}{\Gamma_1\Gamma_2}$	$\frac{C_2}{\Gamma_0}$

$$P_{1,out} \approx \frac{C_3}{\Gamma_0\Gamma_1} + \frac{C_3}{\Gamma_0\Gamma_3}, \quad (22)$$

where $C_3 = 2\ln(2) - 1$. Index 1 indicates a single deployed relay. The second relay in Fig. 1 is removed. In this study we compare a LFOR system with a LFTR system and investigate whether the LFTR system has advantages using the same transmit power budget (cf. Sec. V).

IV. Optimal Power Allocation

The optimal power allocation maximizes the Slepian-Wolf admissible rate region and consequently minimizes the OP. The total transmit power budget E_{T} is redistributed between source and relay(s) to reduce the OP. The transmit power is allocated by $\alpha_i \in [0, 1]$ and $\sum_{i=0}^{M} \alpha_i = 1$, with $M = 1$ for LFOR and $M = 2$ for LFTR, to the corresponding source and relay.

$$E_i = \alpha_i E_{\mathrm{T}} \quad (23)$$

Consequently, the average SNR can be replaced by

$$\Gamma_i = \alpha_j E_{\mathrm{T}} G_i \quad (24)$$

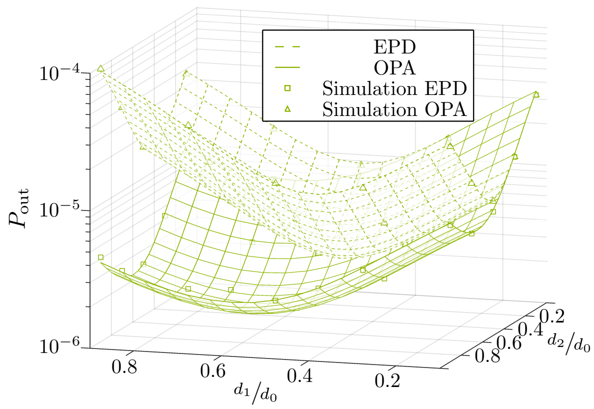


Fig. 3: OP for LFTR system with OPA and EPD at $E_{\text{T}}/N_0 = 15$ dB.

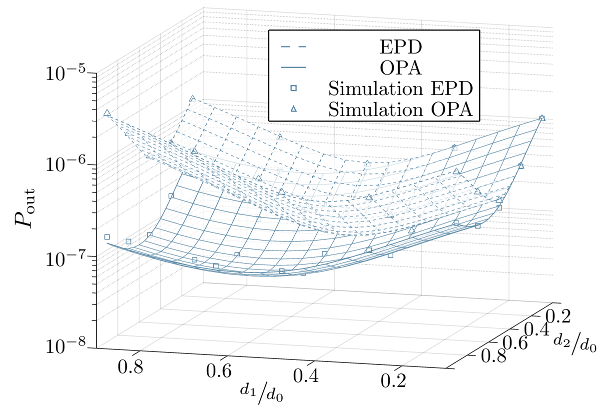


Fig. 4: OP for LFTR system with OPA and EPD at $E_{\text{T}}/N_0 = 20$ dB.

with $(i,j) \in \{(0,0),(1,0),(3,1)\}$ for the LFOR system, $(i,j) \in \{(0,0),(1,0),(2,0),(3,1),(4,2)\}$ for the LFTR system and normalized noise variance N_0 to unity for all channels. Finally, we can formulate optimization problems for the LFTR and LFOR system.

A. LFTR system

Substituting Eq. (24) into Eq. (21) we obtain a *posynomial function* [13, Chap. 4.5]

$$
\begin{aligned}
P_{2,out} \approx & \frac{C_1}{E_{\text{T}}^3 \alpha_0 \alpha_1 \alpha_2 G_0 G_3 G_4} + \frac{C_1}{E_{\text{T}}^3 \alpha_0^2 \alpha_2 G_0 G_2 G_3} \\
& + \frac{C_1}{E_{\text{T}}^3 \alpha_0^2 \alpha_1 G_0 G_1 G_4} + \frac{C_2}{E_{\text{T}}^3 \alpha_0^3 G_0 G_1 G_2}, \qquad (25)
\end{aligned}
$$

and therefore the minimization of the OP can be performed by geometric programming. It has been shown in [13, Chap. 4.5] that any geometric program can be transformed to a convex optimization problem,

$$
\begin{aligned}
\text{minimize} \quad & P_{2,\text{out}}(\alpha_0, \alpha_1, \alpha_2) \\
\text{subject to} \quad & 0 \leq \alpha_i \leq 1, \forall i \qquad (26) \\
& \textstyle\sum_i \alpha_i = 1.
\end{aligned}
$$

B. LFOR system

Substituting Eq. (24) into (22) we obtain a *posynomial function* [13, Chap. 4.5]

$$
P_{1,\text{out}} \approx \frac{C_3}{E_{\text{T}}^2 \alpha_0^2 G_0 G_1} + \frac{C_3}{E_{\text{T}}^2 \alpha_0 \alpha_1 G_0 G_3}. \qquad (27)
$$

Similar to the LFTR system we can formulate a convex optimization problem as shown in [6] to

$$
\begin{aligned}
\text{minimize} \quad & P_{1,\text{out}}(\alpha_0, \alpha_1) \\
\text{subject to} \quad & 0 \leq \alpha_i \leq 1, \forall i \qquad (28) \\
& \textstyle\sum_i \alpha_i = 1.
\end{aligned}
$$

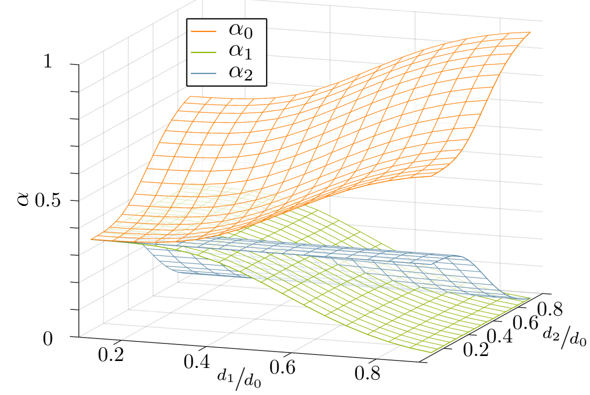


Fig. 5: Power allocation for LFTR system

V. Simulation Results

The convex optimization problems (26) and (28) are numerically solved with *CVX* [14]. All relays are located between the source and the destination. The optimal power allocation (OPA) is compared to a equal power distribution (EPD), where the same transmit power is allocated to the source and relay(s). The approximation of the OP is verified with a Monte-Carlo simulation.

Figs. 3 and 4 present the OP for an LFTR system with $E_{\text{T}}/N_0 = 15$ dB and $E_{\text{T}}/N_0 = 20$ dB, respectively. A significant reduction of the OP for $0.4 \leq d_i/d_0 \leq 0.9, i = 1,2$ with OPA with respect to EPD can be achieved. A maximum reduction of 1.5 orders of magnitude can be achieved at $d_1/d_0 = d_2/d_0 = 0.9$. Fig. 5 presents the optimal allocation for the LFTR system. If the relays are close to the source, the transmit power budget is equally distributed. If the relays are closer to the destination, more power is allocated to the source.

Fig. 6 presents the OP for the LFOR system and a total power budget $E_{\text{T}}/N_0 = 15$ dB and $E_{\text{T}}/N_0 = 20$ dB. Fig. 7 presents the power allocation for the source. If the relay is close to the source, the total transmit power is shared equally between source and relay. The closer the relay gets

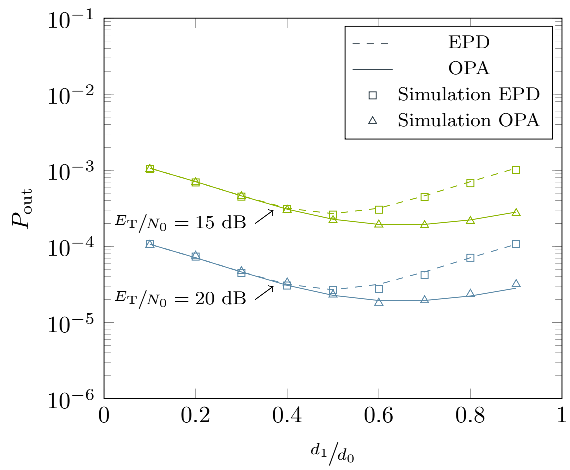


Fig. 6: OP for LFOR system with OPA and EPD at $E_T/N_0 = 15$ dB and $E_T/N_0 = 20$ dB.

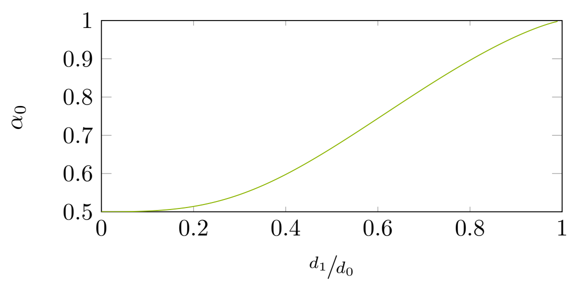


Fig. 7: Source power allocation for LFOR system.

to the destination, the more transmit power is distributed to the source. A reduction of the OPs can be achieved for $0.4 \leq d_1/d_0 \leq 0.9$. The maximum gain of 0.8 orders of magnitude is at $d_1/d_0 = 0.9$.

The advantage of the LFTR system is higher in comparison with the LFOR system. For a total transmit power budget $E_T = 15$ dB or $E_T = 20$ dB a minimum reduction of one order of magnitude up to a maximum reduction of two orders of magnitude for the OP can be achieved.

The maximum relative approximation error propagation can be found for $d_1/d_0 = d_2/d_0 = 0.9$, because the scenario probability $P_{S,4}$ is maximal at this very situation. It is more likely that both relays observe an error if they are far away from the source and consequently the relative approximation error of (16) has the biggest impact. Simulation results in Fig. 3 and Fig. 4 show a maximum relative approximation error propagation that merely totals to 15 % at $d_1/d_0 = d_2/d_0 = 0.9$. For $d_1/d_0 < 0.8$ and $d_2/d_0 < 0.8$ the relative approximation error propagation is negligible. Consequently, the approximation (21) is a good fit for the analytical OP calculation and the desired power allocation.

VI. Conslusion

In this paper we have derived the OP for a LFTR system and presented a power allocation strategy to minimize the OP. First, we have determined the integral for the OP based on the Slepian-Wolf source correlation theorem. Since an exact closed-form expression of the OP cannot be easily achieved, we have investigated the high-SNR behavior and solved the integral for the OP by an approximation. We have also proposed an OPA strategy based on convex optimization to minimize the OP for high-SNR behavior. A significant reduction of the OP with the OPA can be achieved by up to 1.5 orders of magnitude compared to EPD. We have compared the performance of the LFTR and the LFOR system. With the same total transmit power budget, the LFTR system outperforms the LFOR system by at least one order of magnitude up to two orders of magnitude.

The present study indicates that *lossy forwarding* tremendously reduces the OP and therefore enhances robustness and energy efficiency of mesh networks.

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