An Efficient Power Allocation Scheme for Multirelay Systems with Lossy Intra-Links

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***Abstract*—The so-called chief executive officer problem suggests that the source message can be recovered at the destination by merging a set of corrupted replicas forwarded by multiple relays, as long as these replicas are sufficiently correlated with the original message. In this work, we build on Slepian-Wolf's correlated source coding theorem to design a simple, yet efficient power allocation scheme for a multirelay system in which the direct link is unavailable to convey information. In such a system, the replicas forwarded by the relays are allowed to contain intra-link errors due to previous unreliable hops, and the destination is supposed to retrieve the source message by jointly decoding all received replicas. Importantly, the proposed power allocation is asymptotically optimal at high signal-to-noise ratio.**

***Index Terms*—CEO problem, distributed source coding, outage probability, power allocation, relay channel, Slepian-Wolf theorem.**

I. Introduction

In conventional decode-and-forward (DF) relaying systems, reliable communication is achieved by means of some error-control mechanism at the relays. Whenever a relay identifies an uncorrectable error, it simply discards the message and invokes a retransmission [1]. However, discarding an erroneous message may be less appropriate and effective than forwarding it to the destination [2]. First, in extreme scenarios such as under severe environmental disasters or terrorist attacks, the communication infrastructure may be seriously damaged. In these cases, intact mobile devices can be used to establish an emergency mesh network, but the resulting source-relay links (a.k.a. intra-links) are unusually weak due to power constraints inherent to those devices, causing errors to be more frequent. Accordingly, the amount of retransmissions required by conventional DF schemes may be too large, if not impracticable. Another application scenario is the harsh and rapidly-changing propagation environment encountered in vehicular networks envisaged for fifth-generation wireless systems. Second and more fundamentally, it has been shown in [2] that the system performance can be indeed improved by allowing the intra-link errors (IE) to be forwarded to the destination. Such a scheme is referred to as DF-IE. The central idea behind this scheme is that erroneous replicas at the relays are still highly correlated with the source message and thus can be jointly decoded to recover the message at the destination, instead of being merely discarded.

Some DF-IE relaying networks have been considered in previous studies. A particular scenario that has been well investigated is the classic three-node relay system, in which one source (S) communicates with one destination (D) not only through a direct link but also with the aid of one intermediate relay (R). A pioneering DF-IE protocol for such system was proposed in [2], based on a distributed source coding (DSC) scheme that exploits the correlation between the source and relay sequences. In that work, the source-destination (SD) and relay-destination (RD) links were modeled as block Rayleigh fading channels, whereas the source-relay (SR) link was modeled as a binary symmetric channel (BSC) with a certain bit-flipping probability [3]. The BSC assumption has arisen as an amalgamated model for lossy multiple hops between the source and relay. At the relay, a possibly erroneous message is detected, re-encoded, interleaved, and forwarded to the destination. Then, at the destination, a joint decoding (JD) algorithm is applied to retrieve the source message from the direct- and relaying-link transmissions, by means of a log-likelihood ratio updating function. A significant improvement of the decoding performance was observed when compared with the conventional DF scheme, in which erroneous messages are thrown away. The performance gain of DF-IE can be reasoned with use of Slepian-Wolf's correlated source coding theorem [4]. Following this approach, in [5], a theoretical analysis of the outage probability and its asymptotic properties was carried out for the DF-IE scheme introduced in [2]. Afterwards, the same authors proposed in [6] a corresponding power allocation strategy that minimizes the outage probability. More recently, a rigorous and comprehensive outage analysis of the DF-IE protocol was performed in [7], overcoming two drawbacks of previous investigations. First, a block Rayleigh fading model was used not only for the SD and RD links but also for the SR link, thereby taking into account the channel fluctuations of all the links. Second, instead of the Slepian-Wolf theorem, the theorem for source coding with side information was adopted as the appropriate theoretical framework to analyze the exact system performance. After all, in such system the destination is ultimately interested in recovering only the source sequence, with the relay sequence being nothing but a helper. On the other hand, it was demonstrated in [7] that the Slepian-Wolf theorem yields a simple, yet accurate approximation to the true performance.

In this work, inspired by the results in [5] and [6], we design

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a simple, yet efficient power allocation policy for a DSC/JD scheme operating over a DF-IE relaying system. Differently from [5] and [6], we consider a less favorable scenario, in which no usable direct link is available between source and destination. This is probably the case in catastrophic environments with a severe damage to the network infrastructure, or in fifth-generation vehicular networks with unfriendly and highly dynamic propagation conditions, causing source and destination to be unusually far apart from each other. In addition, more generally than those works, we consider that each message is simultaneously transmitted along an arbitrary number of relay routes (vs. a single relay route). Once again, this is probably the case in such abnormal environments, in which these many routes may be required for keeping connectivity at an acceptable level. Similarly to [5] and [6], we consider that the links between the source and the relays are modeled as independent BSCs. It was shown in [8] that a JD scheme at the destination can exploit the correlation among the replicas received from multiple relays, and that a significant performance gain can be attained compared to conventional coding schemes. However, error-free retrieval of the original message cannot be achieved [9]. This is known as the chief executive officer (CEO) problem in network information theory [10]. A general exact solution to the system performance is rather intricate and still open. In particular, for an insightful bound analysis of the Hamming distortion with two relays, we refer the reader to [11]. Herein, we relax the problem by conveniently defining a certain outage event and by computing its probability, the minimization of which eventually enables us to conceive an efficient power allocation scheme. As in [6], we capitalize on the Slepian-Wolf theorem. This is a reasonable framework, since the various relay sequences can be regarded as mutually correlated sources of information. However, we show that the scope of that theorem must be suitably modified to better comply with the requirements of our application. Based on this modified framework, we obtain a single-fold integral-form expression for the outage probability of the particular case with two relays. More importantly, we obtain a useful closed-form asymptotic outage expression for the general case with an arbitrary number of relays. Finally, from this result, we derive a general, remarkably simple power allocation strategy that is asymptotically optimal at high signal-to-noise ratio (SNR). To the best of our knowledge, Slepian-Wolf-based outage analysis and power allocation design for DF-IE relaying networks have not been addressed yet in the context of multiple relays or unavailable direct transmission.

An important remark is in order. As shown in [7], although the Slepian-Wolf theorem serves as an useful analytical framework to assess the performance of DF-IE systems, such framework is an approximate one, since the destination is primarily interested in recovering the source sequence, and not the relay sequences. Hence, any outage analysis based on the Slepian-Wolf theorem is inherently approximate, in the sense that the associated outage and non-outage events do not rigorously establish the recoverability conditions of the source message. On the other hand, our primary aim here is far from developing a rigorous performance analysis, but a tractable and suitable one that is ultimately employed to devise an efficient power allocation strategy for the investigated system. So we have opted for the Slepian-Wolf framework. As shall be seen, we have tested our power allocation strategy into the practical DSC/JD scheme introduced in [2]. Strikingly, in all the tests, the observed bit-error rate (BER) was nearly optimal.

In what follows, $\Pr[\cdot]$ denotes probability, $f_X(\cdot)$ is the probability density function (PDF) of a continuous random variable X, $p_Y(\cdot)$ is the probability mass function (PMF) of a discrete random variable Y, $H(Y)$ is the entropy of Y, z is a sample realization of a generic random variable Z, $h_b(x) \triangleq -x\log_2(x) - (1-x)\log_2(1-x)$ is the binary entropy function, $|\mathcal{S}|$ is the cardinality of a set $\mathcal{S}$, $\mathcal{B} = \{0,1\}$ is a binary set, and $\{A_i | i \in \mathcal{S}\}$ is an indexed series (e.g., $\{A_i | i \in \{1,5,7\}\} = \{A_1, A_5, A_7\}$).

II. System Model

We consider a half-duplex dual-hop[1] relay system as shown in Fig. 1. It consists of one source S, one destination D, and N DF relays $\mathrm{F}_1, \mathrm{F}_2, \ldots, \mathrm{F}_N$[2]. The system operation is based on the CEO problem. An i.i.d. binary sequence[3] B_0 is originated by S with uniform probabilities $\Pr[B_0 = 0] = \Pr[B_0 = 1] = 0.5$. The source sequence is transmitted via N independent BSCs with associated memoryless binary error sequences E_i, $i \in \{1, \ldots, N\}$—a representation of the accumulated error caused by the multiple wireless hops up to the last one. The PMF of E_i can be written as

$$p_{E_i}(e) = p_i \delta(e-1) + (1-p_i)\delta(e), \tag{1}$$

$e \in \{0,1\}$, in which $0 < p_i \leq 0.5$ is the bit-flipping probability and $\delta(\cdot)$ is the discrete delta function. Therefore, the ith relay F_i observes a sequence $B_i = B_0 \oplus E_i$, $i \in \{1, \ldots, N\}$, with "$\oplus$" denoting the binary exclusive OR operation. Note that, like the source sequence, all relay sequences are also uniformly distributed, so that $H(B_i) = 1$, $i \in \{0, 1, \ldots, N\}$. In addition, note that the relay sequences $B_1, \ldots, B_N$ are mutually correlated. Those sequences are transmitted to D over independent channels undergoing flat Rayleigh fading (RF) and additive white Gaussian noise with mean power N_0. At the destination, the relay sequences are estimated as $\hat{B}_1, \ldots, \hat{B}_N$ and, based on these, the source sequence is finally estimated as $\hat{B}_0$. The PDF of the received instantaneous SNR Γ_i at the ith second hop is exponentially distributed, thus given by

$$f_{\Gamma_i}(\gamma_i) = \frac{1}{\bar{\Gamma}_i} \exp(-\frac{\gamma_i}{\bar{\Gamma}_i}), \tag{2}$$

where $\bar{\Gamma}_i$ is the average SNR, obtained as

$$\bar{\Gamma}_i = (P_i/N_0) \cdot d_i^{-\eta}, \tag{3}$$

with P_i being the transmit power at F_i, d_i being the distance between F_i and D, and η being the pathloss exponent.

[1] Recall that in our model the first hop is an amalgamated representation of possibly multiple hops between the source and each relay.

[2] The relays have been so denoted to avoid confusion with the transmission rates introduced in the next section.

[3] In order to alleviate the notation, we shall drop the time index when denoting data and error sequences.

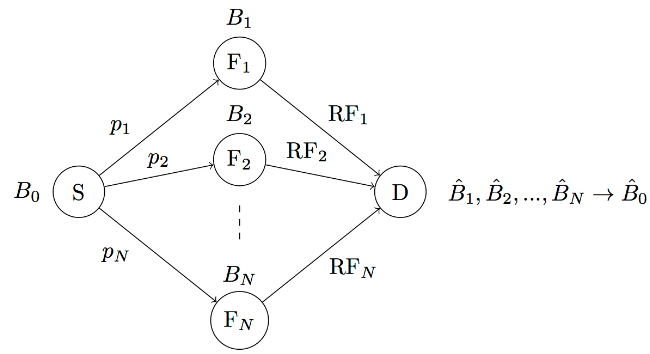


Fig. 1: System model of a multirelay scheme based on the CEO problem.

III. Preliminaries

Of course, the best recovery of the source message B_0 at the destination is expected to be achieved when all recovered relay messages $\hat{B}_1, \ldots, \hat{B}_N$ are error-free. However, even in such a favorable scenario at the second hops, the source error probability $\Pr[\hat{B}_0 \neq B_0]$ cannot be zero, because the formulation of the CEO problem excludes error-free BSCs at the first hops ($p_i > 0, \forall i$). So far, only a few DSC/JD schemes for the referred problem have been proposed that exchange decoding information among the relay sequences as a means to reduce the source error probability. In particular, the joint decoder proposed in [12] shows a significant performance gain when compared to other coding schemes. An exact error-rate calculation for that joint decoder was provided in [9].

Herein, our aim is not to analyze the source error-rate performance of any particular DSC/JD scheme for DF-IE relaying networks. Instead, our aim is to design a general power allocation strategy that can be successfully employed to improve the performance of any such scheme. To this end, from an information-theoretical viewpoint, the system performance can be reasoned by means of the Slepian-Wolf correlated source coding theorem [4], because the multiple relay messages can be regarded as correlated information sources, each of which resembles to some extent the original source message.

A. Slepian-Wolf Theorem: the original scope

The Slepian-Wolf theorem states that iff the transmission rates R_i at the relays, $i \in \{1, \ldots, N\}$, measured in bits per channel use, satisfy the inequality constraints [4]

$$\begin{aligned}\sum_{i \in \mathcal{S}} R_i \geq & H\left(\{B_i | i \in \mathcal{S}\} | \{B_j | j \in \mathcal{S}^c\}\right) \\ = & H\left(B_1, \ldots, B_N\right) - H\left(\{B_j | j \in \mathcal{S}^c\}\right) \end{aligned} \tag{4}$$

for all subsets $\mathcal{S} \subseteq \{1, \ldots, N\}$, then all the relay sequences $B_1, \ldots, B_N$ can be recovered error-free, with $\mathcal{S}^c$ denoting the complement of $\mathcal{S}$. The set of N-tuples $R_1, \ldots, R_N$ that satisfy all the constraints in (4) is referred to as the Slepian-Wolf admissible rate region. We now find this region in terms of the bit-flipping probabilities of the first hops. Note in (4) that each constraint is written in terms of (i) the joint entropy of all the relay sequences and (ii) the joint entropy of a certain subset $\{B_j | j \in \mathcal{S}^c\}$ of relay sequences. Any of these entropies can be evaluated as special cases of this formula:

$$\begin{aligned} H(\{B_i | i \in S\}) = -\sum_{\{b_i\} \in \mathcal{B}^{|S|}} & \Pr\left[\{B_i | i \in S\} = \{b_i\}\right] \\ & \times \log_2\left(\Pr\left[\{B_i | i \in S\} = \{b_i\}\right]\right). \end{aligned} \tag{5}$$

The required probabilities $\Pr\left[\{B_i | i \in S\} = \{b_i\}\right]$ can be obtained by knowing that the source bits are equally likely and by recognizing that both $B_0 = 0$ and $B_0 = 1$ may lead to each possible sample realization of $\{B_i | i \in S\}$. This gives

$$\Pr\left[\{B_i | i \in S\} = \{b_i\}\right] = \frac{1}{2}\left[\prod_{i \in \mathcal{S}} p_{E_i}(b_i) + \prod_{i \in \mathcal{S}} \bar{p}_{E_i}(b_i)\right], \tag{6}$$

where we have used (i) the independence among the error sequences and (ii) the auxiliary PMF

$$\bar{p}_{E_i}(e) \triangleq (1 - p_i)\delta(e - 1) + p_i \delta(e), \tag{7}$$

defined by swapping the probabilities of E_i. Note that (6) is ultimately given in terms of the bit-flipping probabilities p_i associated with the first hops. Accordingly, using this into (5) and then into (4), we obtain each rate constraint of the Slepian-Wolf theorem also in terms of these bit-flipping probabilities.

B. Slepian-Wolf Theorem: a modified scope

In its original scope, the Slepian-Wolf theorem provides the rate conditions for recovering all relay messages at the destination. However, from an engineering perspective, this is not the primary aim. In the investigated system, the destination is really not interested in recovering all relay messages—which are possibly erroneous, indeed—, but in merging them somehow to recover the original source message. Note that each relay sequence contains a different amount of information about the source sequence, depending on the channel quality of the first hop. To gain insight, let us consider two extreme situations. First, when a first hop is fully unreliable, i.e., if its bit-flipping probability equals 0.5, then that relay sequence cannot contain any useful information about the source sequence and should be just discarded. In such case, it would be nonsense to impose any rate constraint on the associated second hop. Second, when a first hop is fully reliable, i.e., if its bit-flipping probability is zero, then that relay sequence actually coincides with the source sequence. In such case, it would be desirable to entirely recover the relay sequence, which calls for a full rate constraint. These two examples suggest that, in the general case, an appropriate rate requirement for a given relay should not depend on the absolute information content of the relay message (i.e., its entropy), but on how much of this content concerns the source message (i.e., its mutual information regarding the source). This can be accomplished by adapting the Slepian-Wolf theorem accordingly. All in all, we propose to modify the original scope of that theorem by replacing each entropy term with a corresponding mutual information term involving the source message. Specifically, the transmission rates R_i at the relays, $i \in \{1, \ldots, N\}$, must satisfy the inequality constraints

$$\sum_{i\in\mathcal{S}} R_i \geq I\left(\{B_i|i\in\mathcal{S}\};B_0|\{B_j|j\in\mathcal{S}^c\}\right) = I\left(B_1,\ldots,B_N;B_0\right) - I\left(\{B_j|j\in\mathcal{S}^c\};B_0\right). \quad (8)$$

Hereafter, the set of N-tuples $R_1,\ldots,R_N$ that satisfy all the constraints in (8) is referred to as the modified Slepian-Wolf admissible rate region. As before, we now find this region in terms of the bit-flipping probabilities of the first hops. This is paramount for many derivations that follow. Note in (8) that each constraint is written in terms of (i) the mutual information between all relay sequences and the source sequence and (ii) the mutual information between a certain subset $\{B_j|j\in\mathcal{S}^c\}$ of relay sequences and the source sequence. Any of these mutual-information terms can be evaluated as special cases of this formula:

$$I(\{B_i|i\in S\};B_0) = H(\{B_i|i\in S\}) - H(B_0,\{B_i|i\in S\}) + 1, \quad (9)$$

where $H(\{B_i|i\in S\})$ is defined as in (5) and

$$H(B_0,\{B_i|i\in S\}) = -\sum_{\{b_0,\{b_i\}\}\in\mathcal{B}^{|\mathcal{S}|+1}} \Pr\left[\{B_0,\{B_i|i\in S\}\}=\{b_0,\{b_i\}\}\right] \times \log_2\left(\Pr\left[\{B_0,\{B_i|i\in S\}\}=\{b_0,\{b_i\}\}\right]\right). \quad (10)$$

By knowing that the source bits are equally likely, the required probabilities $\Pr\left[\{B_0,\{B_i|i\in S\}\}=\{b_0,\{b_i\}\}\right]$ can be obtained as

$$\Pr\left[\{B_0,\{B_i|i\in S\}\}=\{b_0,\{b_i\}\}\right] = \frac{1}{2}\left[\delta(b_0)\prod_{i\in\mathcal{S}} p_{E_i}(b_i) + \delta(b_0-1)\prod_{i\in\mathcal{S}}\bar{p}_{E_i}(b_i)\right]. \quad (11)$$

Note that (11) is ultimately given in terms of the bit-flipping probabilities p_i associated with the first hops. Accordingly, using this into (10) and then into (9) and (8), we obtain each rate constraint of the modified Slepian-Wolf theorem also in terms of these bit-flipping probabilities. Next we illustrate this process for the particular case of two relays.

IV. Outage Probability

In the proposed system, an outage event occurs whenever the transmission rates $R_1,\ldots,R_N$ fall outside the modified Slepian-Wolf admissible rate region. This condition means that, at least for one of the relays, its information content regarding the source message cannot be entirely recovered at the destination. The maximum achievable value of R_i is related to the received SNR Γ_i by means of [7]

$$R_i = \frac{1}{R_{ci}}\log_2\left(1+\Gamma_i\right), \quad (12)$$

where R_{ci} represents the spectrum efficiency associated with the modulation and channel coding schemes [6]. In many parts of this work, for simplicity, we shall assume $R_{ci}=R_c, \forall i$. Using (12), each rate constraint in (8) that defines an outage event can be mapped into an equivalent SNR constraint. In this section, we follow this approach to derive an exact integral-form expression for the outage probability of the particular case with two relays. More importantly, we derive a simple and useful closed-form asymptotic outage expression for the general case with an arbitrary number of relays, in which an exact solution proves intractable. In the next section, this general expression shall be the basis for the design of a power allocation scheme.

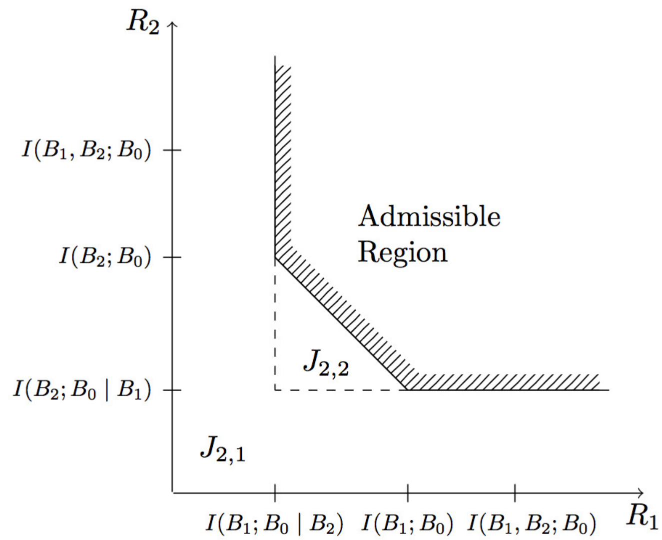


Fig. 2: Modified Slepian-Wolf inadmissible rate region for two relays.

A. Two Relays

Fig. 2 shows the modified inadmissible rate region for two relays[4]. It is divided into two areas, with associated probabilities $J_{2,1}$ and $J_{2,2}$. Thus, the outage probability P_{out} for two relays can be formulated as

$$P_{\text{out}} = J_{2,1} + J_{2,2}. \quad (13)$$

Substituting (12) into the rate inequalities in (8), $J_{2,1}$ and $J_{2,2}$ can be expressed in terms of SNR constraints as

$$\begin{aligned} J_{2,1} &= 1 - \Pr\left[R_1 > I\left(B_1;B_0|B_2\right), R_2 > I\left(B_2;B_0|B_1\right)\right] \\ &= 1 - \Pr\Big[2^{R_{c1}I(B_1;B_0|B_2)} - 1 < \Gamma_1 < \infty, \\ &\qquad 2^{R_{c2}I(B_2;B_0|B_1)} - 1 < \Gamma_2 < \infty\Big], \end{aligned} \quad (14)$$

$$\begin{aligned} J_{2,2} &= \Pr\left[I\left(B_1;B_0|B_2\right) < R_1 < I\left(B_1;B_0\right),\right. \\ &\qquad \left. I\left(B_2;B_0|B_1\right) < R_2 < I\left(B_1,B_2;B_0\right) - R_1\right] \\ &= \Pr\Big[2^{R_{c1}I(B_1;B_0|B_2)} - 1 < \Gamma_1 < 2^{R_{c1}I(B_1;B_0)} - 1, \\ &\qquad 2^{R_{c2}I(B_2;B_0|B_1)} - 1 < \Gamma_2 < \\ &\qquad 2^{R_{c2}I(B_1,B_2;B_0) - \frac{R_{c2}}{R_{c1}}\log_2(\Gamma_1+1)} - 1\Big]. \end{aligned} \quad (15)$$

These expressions can be evaluated by integrating the joint PDF $f_{\Gamma_1,\Gamma_2}(\gamma_1,\gamma_2) = f_{\Gamma_1}(\gamma_1)f_{\Gamma_2}(\gamma_2)$ over the corresponding

[4]Note that $I(B_1;B_0|B_2) < I(B_1;B_0)$ and $I(B_2;B_0|B_1) < I(B_2;B_0)$, because $B_1 \rightarrow B_0 \rightarrow B_2$ form a Markov chain.

$$
\begin{aligned}
P_{\text{out}} =& 1-\exp\left(-\frac{2^{R_{c1}I(B_1;B_0|B_2)}-1}{\bar{\Gamma}_1}-\frac{2^{R_{c2}I(B_2;B_0|B_1)}-1}{\bar{\Gamma}_2}\right)+\frac{1}{\bar{\Gamma}_1}\exp\left(-\frac{2^{R_{c2}I(B_2;B_0|B_1)}-1}{\bar{\Gamma}_2}\right)\\
&\times\int_{2^{R_{c1}I(B_1;B_0|B_2)}-1}^{2^{R_{c1}I(B_1;B_0)}-1} e^{-\frac{\gamma_1}{\bar{\Gamma}_1}}\left[1-\exp\left(\frac{2^{R_{c2}I(B_1,B_2;B_0)-\frac{R_{c2}}{R_{c1}}\log_2(1+\gamma_1)}}{\bar{\Gamma}_2}+\frac{2^{R_{c2}I(B_2;B_0|B_1)}}{\bar{\Gamma}_2}\right)\right]d\gamma_1, \qquad (16)
\end{aligned}
$$

ranges defined in (14) and (15). This is done in Appendices A and B. By combining the results therein, P_{out} can be finally written in exact single-fold integral form as in (16), shown at the top of the next page. where $I(B_1;B_0) = 1 - h_b(p_1)$, and $I(B_1;B_0|B_2)$, $I(B_2;B_0|B_1)$, and $I(B_1,B_2;B_0)$ are obtained from (8)–(11) in terms of the bit-flipping probabilities p_1 and p_2 as

$$
\begin{aligned}
I(B_1;B_0|B_2) &= h_{21}(p_1,p_2) - h_{22}(p_1,p_2) + h_b(p_2),\\
I(B_2;B_0|B_1) &= h_{21}(p_1,p_2) - h_{22}(p_1,p_2) + h_b(p_1), \qquad (17)\\
I(B_1,B_2;B_0) &= h_{21}(p_1,p_2) - h_{22}(p_1,p_2) + 1,
\end{aligned}
$$

where

$$h_{21}(p_1,p_2) \triangleq -2\sum_{i=1}^{2} a_{21}(i)\log_2(a_{21}(i)), \qquad (18)$$

with

$$
\begin{aligned}
a_{21}(1) &\triangleq 0.5[p_1p_2 + (1-p_1)(1-p_2)],\\
a_{21}(2) &\triangleq 0.5[p_1(1-p_2) + (1-p_1)p_2],
\end{aligned} \qquad (19)
$$

and

$$h_{22}(p_1,p_2) \triangleq -2\sum_{i=1}^{4} a_{22}(i)\log_2(a_{22}(i)), \qquad (20)$$

with

$$
\begin{aligned}
a_{22}(1) &\triangleq 0.5[p_1p_2],\\
a_{22}(2) &\triangleq 0.5[p_1(1-p_2)],\\
a_{22}(3) &\triangleq 0.5[(1-p_1)p_2],\\
a_{22}(4) &\triangleq 0.5[(1-p_1)(1-p_2)].
\end{aligned} \qquad (21)
$$

Although the outage expression in (16) cannot be solved in exact closed form, a simple asymptotic solution can be derived at high SNR. This is also done in Appendices A and B for $J_{2,1}$ and $J_{2,2}$, respectively, by assuming $R_{c1} = R_{c2} = R_c$. From the results therein, it turns out that the diversity order of $J_{2,2}$ is greater than that of $J_{2,1}$, so that the latter dominates the high-SNR outage behavior. Accordingly, an asymptotic expression of P_{out} for two relays can be written in compact form as

$$P_{\text{out}} \simeq \frac{C_1}{\bar{\Gamma}_1} + \frac{C_2}{\bar{\Gamma}_2}, \qquad (22)$$

where the constants C_1 and C_2 are defined as

$$C_1 \triangleq 2^{R_cI(B_1;B_0|B_2)} - 1, \qquad (23)$$

$$C_2 \triangleq 2^{R_cI(B_2;B_0|B_1)} - 1, \qquad (24)$$

with $I(B_1;B_0|B_2)$ and $I(B_2;B_0|B_1)$ being given in (17) in terms of the bit-flipping probabilities p_1 and p_2.

B. N Relays

The same approach can be applied to an arbitrary number N of relays, by splitting the modified inadmissible rate region into several parts. The contribution of each part is then evaluated by integrating the joint PDF of the individual SNRs over the corresponding range. However, as in the case of two relays, the resulting outage expression is written in $(N-1)$-fold integral form. On the other hand, here again, a simple closed-form asymptotic solution at high SNR can be also obtained for the general case with an arbitrary number of relays. This is based on the following key result of a pioneering work in [13]: the asymptotic outage behavior at high SNR is exclusively determined by the PDF behavior of the SNR in the vicinity of the origin. In our case, it suffices to consider those parts of the modified inadmissible rate region that directly interface with at least one of the coordinate axes. From the Slepian-Wolf constraints given in (8), the probability mass $J_{N,1}$ of the referred parts can be expressed as

$$
\begin{aligned}
J_{N,1} =& 1 - \Pr\left[R_1 > I(B_1;B_0|B_2,\ldots,B_N),\right.\\
&R_2 > I(B_2;B_0|B_1,\ldots,B_N),\ldots,\\
&\left.R_N > I(B_N;B_0|B_1,\ldots,B_{N-1})\right],\\
=& 1 - \Pr\left[2^{R_{c1}I(B_1;B_0|B_2,\ldots,B_N)} - 1 < \Gamma_1 < \infty,\right.\\
&2^{R_{c2}I(B_2;B_0|B_1,\ldots,B_N)} - 1 < \Gamma_2 < \infty,\ldots,\\
&\left.2^{R_{cN}I(B_N;B_0|B_1,\ldots,B_{N-1})} - 1 < \Gamma_N < \infty\right].
\end{aligned} \qquad (25)
$$

Now, by following the same procedure presented in Appendices A and B for two relays, after some algebraic manipulations, an asymptotic high-SNR expression for the outage probability P_{out} of the general case with an arbitrary number of relays can be finally obtained as

$$P_{\text{out}} \simeq \sum_{i=1}^{N}\frac{C_i}{\bar{\Gamma}_i}, \qquad (26)$$

where each constant C_i is defined as

$$C_i \triangleq 2^{R_cI(B_i;B_0|\{B_j,j\neq i\})} - 1, \qquad (27)$$

with the mutual-information terms $I(B_i;B_0|\{B_j, j \neq i\})$, $i \in \{1,\ldots,N\}$, being computed from (8)–(11) in terms of the bit-flipping probabilities $p_1,\ldots,p_N$.

C. Throughput

Although the outage probability is an effective measure for the likelihood that each transmission succeeds, it does not capture how much useful information the destination receives on average per transmission. To capture this, following the standard approach in the literature, we define the system throughput T as the mutual information between the set of relay sequences $B_1, \ldots, B_N$ and the source sequence B_0 times the non-outage probability, which gives

$$T = I(B_1, \ldots, B_N; B_0) \cdot (1 - P_{\text{out}}), \tag{28}$$

where $I(B_1, \ldots, B_N; B_0)$ and P_{out} can be computed by using the mathematical framework developed heretofore.

It is noteworthy that increasing the number of relays asks for extra time slots or extra frequency channels to accommodate the entire relaying traffic. In other words, as the number of relays increases, the cooperative diversity increases as well, but the spectral efficiency tends to diminish. This is a very important trade-off that must be taken into account in any practical cooperative design. We do not further elaborate on this topic in this work, as our aim is not to provide a comprehensive study of spectral efficiency for the investigated system, but to provide an efficient power allocation scheme that maximizes the noise immunity, i.e., that minimizes the bit error probability. Accordingly, our definition of system throughput in (28) reflects the average flow of information (bits) not *per second per Hertz*, but *per transmission*—regardless of the amount of spectrum each transmission process may require.

V. Asymptotically Optimal Power Allocation

In this section we design a simple power allocation strategy for the multiple relays in order to improve the outage performance of the investigated system. Despite its simplicity, the proposed allocation proves highly effective, being asymptotically optimal at high SNR. For that reason, we call it Asymptotically Optimal Power Allocation (AOPA).

Given a total amount of transmit power P_T for all relays, the transmit power at the ith relay is assigned as $P_i = \alpha_i P_T$, where $0 \le \alpha_i \le 1$ is the power allocation coefficient, $i \in \{1, \ldots, N\}$. Of course, $\sum_{i=1}^{N} \alpha_i = 1$. Then, from (3), the average received SNR at the ith second hop can be written as

$$\bar{\Gamma}_i = \frac{\alpha_i \, P_{\text{T}} \, d_i^{-\eta}}{N_0}. \tag{29}$$

Our primary aim is to find the set of power allocation coefficients $\alpha_1, \ldots, \alpha_N$ that minimize P_{out}, that is,

$$\begin{aligned} &\underset{\alpha_1,\ldots,\alpha_N}{\text{minimize}} \quad P_{\text{out}}(\alpha_1, \ldots, \alpha_N) \\ &\text{subject to} \quad 0 \le \alpha_i \le 1, \forall i, \text{ and } \sum_{i=1}^{N} \alpha_i = 1. \end{aligned} \tag{30}$$

Unfortunately, as seen in the previous section, there exists no general exact closed-form expression for P_{out}. Instead, we propose to minimize the simple asymptotic outage expression in (26). By using (3), this can be formulated as

$$\begin{aligned} &\underset{\alpha_1,\ldots,\alpha_N}{\text{minimize}} \quad \sum_{i=1}^{N} \frac{N_0 C_i d_i^{\eta}}{P_{\text{T}}} \cdot \frac{1}{\alpha_i} \\ &\text{subject to} \quad 0 \le \alpha_i \le 1, \forall i, \text{ and } \sum_{i=1}^{N} \alpha_i = 1, \end{aligned} \tag{31}$$

where each constant C_i is defined as in (27). This is a convex optimization problem, as follows. Note that the cost function is a summation, each component of which being a function of a single power allocation coefficient. It turns out that the ith component $N_0 C_i d_i^{\eta}/(P_{\text{T}}\alpha_i)$ is a convex function of the ith coefficient α_i, because $N_0 C_i d_i^{\eta}/P_{\text{T}} \ge 0$ and $1/\alpha_i$ is a convex function of α_i. The proof of convexity is completed by recognizing that a sum of convex functions is also a convex function [14]. To find its global minimum, we eliminate the Nth power allocation coefficient α_N by incorporating the constraint $\sum_{i=1}^{N} \alpha_i = 1$ into the cost function, which gives

$$\sum_{i=1}^{N-1} \frac{N_0 C_i d_i^{\eta}}{P_{\text{T}}} \cdot \frac{1}{\alpha_i} + \frac{N_0 C_N d_N^{\eta}}{P_{\text{T}}} \cdot \frac{1}{1 - \sum_{i=1}^{N-1} \alpha_i}. \tag{32}$$

Then, by differentiating (32) with respect to the remaining set of power allocation coefficients $\alpha_1, \ldots, \alpha_{N-1}$, by equating all these partial derivatives to zero, and by solving the resulting system of equations, after some algebraic manipulations omitted here for simplicity, we finally arrive at the AOPA scheme:

$$\alpha_i^* = \frac{\sqrt{C_i \cdot d_i^{\eta}}}{\sum_{j=1}^{N} \sqrt{C_j \cdot d_j^{\eta}}}, \; i \in \{1, \ldots, N\}. \tag{33}$$

Note that the proposed power allocation depends ultimately on the distances d_i between each relay and the destination, the path loss exponent η, and the conditional mutual-information terms $I(B_i; B_0 | \{B_j, j \ne i\})$ between each relay sequence and the source sequence, which, in turn, are provided in (8)–(11) in terms of the bit-flipping probabilities of the first hops. Also note that the solution in (33) inherently complies with the constraint $0 \le \alpha_i \le 1$. This is the main analytical contribution of this work.

VI. SNR Gain

In order to examine the SNR gain provided by AOPA ($\alpha_i = \alpha_i^*$) vs. Equal Power Allocation (EPA), i.e., $\alpha_i = 1/N$, we consider the corresponding asymptotic reduction in SNR while achieving the same outage probability. This is normally expressed in dB units, as follows:

$$G_{[\text{dB}]} = \left[(P_{\text{T}}/N_0)_{\text{EPA, [dB]}} - (P_{\text{T}}/N_0)_{\text{AOPA, [dB]}} \right]_{P_{\text{out}} \text{ (fixed)}}. \tag{34}$$

Based on (26) and (29), the average transmit SNR of the two allocation schemes can be asymptotically expressed as

$$\begin{aligned} (P_{\text{T}}/N_0)_{\text{EPA}} &= N \sum\nolimits_i C_i \cdot d_i^{\eta}/P_{\text{out}} \ge (P_{\text{T}}/N_0)_{\text{AOPA}} \\ &= \sum\nolimits_i C_i \cdot d_i^{\eta} / (\alpha_i^* P_{\text{out}}). \end{aligned} \tag{35}$$

From (33) and (VI), we can finally evaluate (34) as

$$G_{[\text{dB}]} = 10 \log_{10} \left(\frac{N \sum_i C_i \cdot d_i^{\eta}}{\left(\sum_i \sqrt{C_i \cdot d_i^{\eta}} \right)^2} \right) \ge 0. \tag{36}$$

Here again, observe that the AOPA-over-EPA SNR gain in (36) depends ultimately on the distances d_i between each relay and the destination, the path loss exponent η, and the bit-flipping probabilities $p_1, \ldots, p_N$ of the first hops.

VII. Practical Application

In order to assess the error-rate performance of our AOPA strategy from a practical viewpoint, we have applied it into the

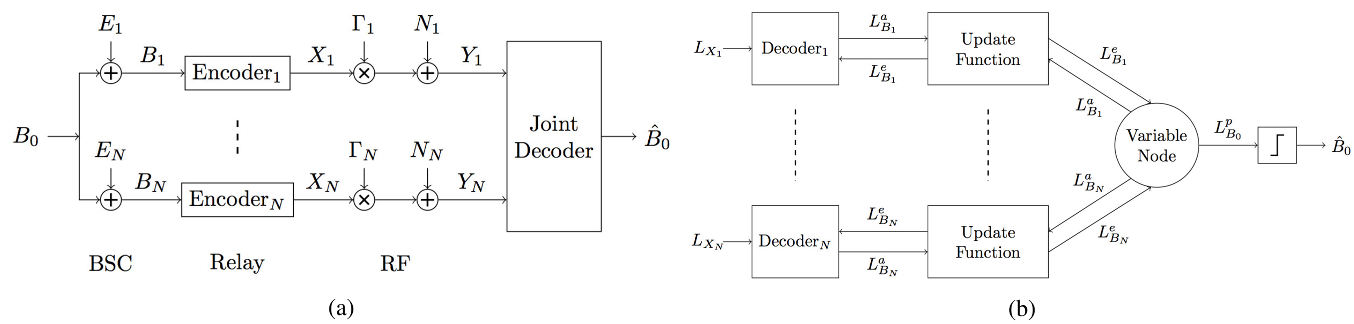


Fig. 3: Block diagram of the DSC/JD scheme: (a) end-to-end system; (b) joint decoder.

DSC/JD scheme introduced in [2]. Fig. 3a shows a block diagram of this scheme, which complies with the DF-IE system model established in Section II. For clarity, we now present a brief description of the transmission scheme in [2]. Each relay sequence B_i is interleaved and encoded by means of a twofold serially concatenated code. First, a systematic non-recursive convolutional code (SNRCC) is deployed, followed by a doped accumulator (ACC), i.e., a memory-1 systematic recursive convolutional code (SRCC). The ACC is used to prevent an error floor at the relay decoder [15]. Then, each coded sequence X_i is transmitted over a block Rayleigh fading channel, with the received instantaneous SNRs Γ_i at the second hops being exponentially distributed, as given in (2) and (3). Finally, at the destination, $\hat{B}_0$ is estimated by the joint decoder, as illustrated in Fig. 3b. A soft demapping is initially applied by calculating the log-likelihood ratio (LLR) L_{X_i} from the received sequence Y_i and known channel state information. Each relay's decoder runs two matching BCJR algorithms [16]. The JD operation is structured in two main stages: a local iteration, where each relay sequence is decoded, and a global iteration, where an exchange of information among all relay sequences is performed. In this second stage, the LLRs $L^e_{B_i}$ are iteratively refreshed by an update function, based on the knowledge of the bit-flipping probabilities $p_1, \ldots, p_N$[5]. Once these LLRs stop changing with the iterations, a final estimation $\hat{B}_0$ is then established by a hard decision over the sum of all LLRs. For more details on this scheme, please refer to [2]. The corresponding source error-rate performance when subject to AOPA shall be presented and discussed in the next section.

VIII. Numerical Results

In this section we evaluate the impact of the our AOPA policy on the performance of the investigated DF-IE relaying system, by considering some representative sample scenarios. The EPA policy, i.e., $\alpha_i = 1/N$, $\forall i$, is included for comparison. In each scenario, the outage probability is assessed in an asymptotic fashion, from (26), as well as via Monte Carlo simulation, whereas the throughput is assessed via simulation only. For illustration purposes, we assume a binary phase-shift keying modulation and a channel-code rate of $1/2$, so that $R_c = 2$. Moreover, we assume $\eta = 4$ and a normalized distance $0 < d_i \leq 1$ between relays and destination. We consider two, three, and four relays under a myriad of configurations for bit-flipping probabilities, relay location, and average SNR. Most investigated scenarios are listed in Table I, along with the power allocation coefficients of AOPA and the corresponding SNR gains with respect to EPA, in terms of both outage probability and average BER (i.e., source error rate). The SNR gain is obtained by (36) for the outage probability and by simulation for BER. Unless otherwise stated, $d_i = 0.5$ for all relays.

Fig. 4 shows the outage probability versus the average system transmit SNR for different numbers of relays under non-identical bit-flipping probabilities. Note that scenarios 1, 3, and 5—as well as scenarios 2, 4, and 6—corresponding to two, three, and four relays, respectively, have been chosen with identical minimum values (best first hops) and identical maximum values (worst first hops) of the bit-flipping probabilities. This is to allow for a fair comparison between different numbers of relays. The following can be observed from the curves: (i) our asymptotic expression in (26) gives an excellent match at medium to high SNR; (ii) in all the cases AOPA outperforms EPA at medium to high SNR; and (iii) the more dissimilar are the bit-flipping probabilities, the greater is the SNR gain achieved by AOPA when compared with EPA. In other words, the SNR gain of AOPA turns out to be smaller when there is inherently less room for improvement by reallocating power among the relays, i.e., when the various relays either have more similar bit-flipping probabilities at the first hops or are located at more similar distances from the destination or both. Note that this is by no means a drawback of the proposed allocation policy, which proved nearly optimal in all scenarios. Instead, this is just a property of the system. That is, depending on the bit-flipping probabilities and distances associated with the various relays, the very performance limit achieved via optimum power allocation happens to be closer to or farther from the performance achieved under equal-power allocation. In particular, when the various relays have identical bit-flipping probabilities and identical distances to destination, equal-power allocation is indeed optimum. In such a case, there is obviously no room for further improvement, causing the SNR gain to be nil.

Fig. 5 depicts the outage probability and throughput versus the average system transmit SNR for two relays, with a constant bit-flipping probability $p_2 = 0.1$ at the second relay and a varying bit-flipping probability $p_1 \in \{0.001, 0.01\}$ at

[5]The bit-flipping probabilities are assumed to be known at the destination.

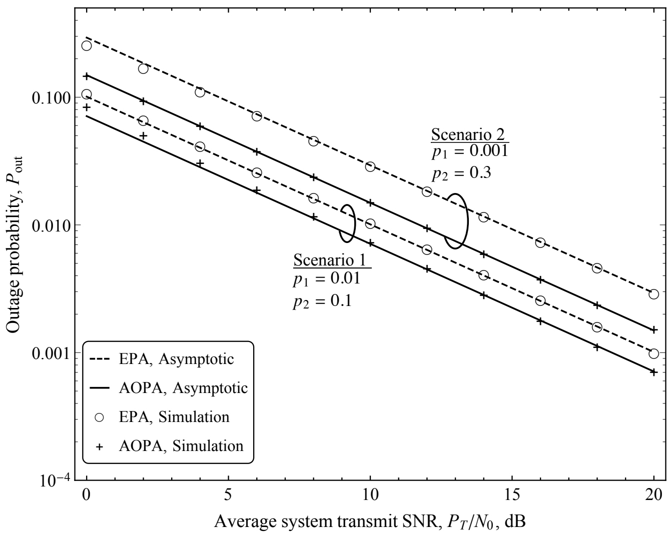
TABLE I: Bit-flipping probabilities, power allocation coefficients, and SNR gains for the investigated scenarios.

Scenario	p_1	p_2	p_3	p_4	α_1	α_2	α_3	α_4	SNR gain (dB) in outage	SNR gain (dB) in BER
1	0.01	0.1	—	—	0.824	0.176	—	—	1.53	0.98
2	0.001	0.3	—	—	0.983	0.017	—	—	2.87	2.83
3	0.01	0.06	0.1	—	0.577	0.232	0.191	—	1.04	0.49
4	0.001	0.06	0.3	—	0.881	0.087	0.032	—	3.72	2.18
5	0.01	0.05	0.06	0.1	0.429	0.218	0.201	0.152	0.73	0.39
6	0.001	0.05	0.06	0.3	0.721	0.122	0.116	0.041	3.43	2.24
7	0.001	0.1	—	—	0.941	0.059	—	—	2.54	—

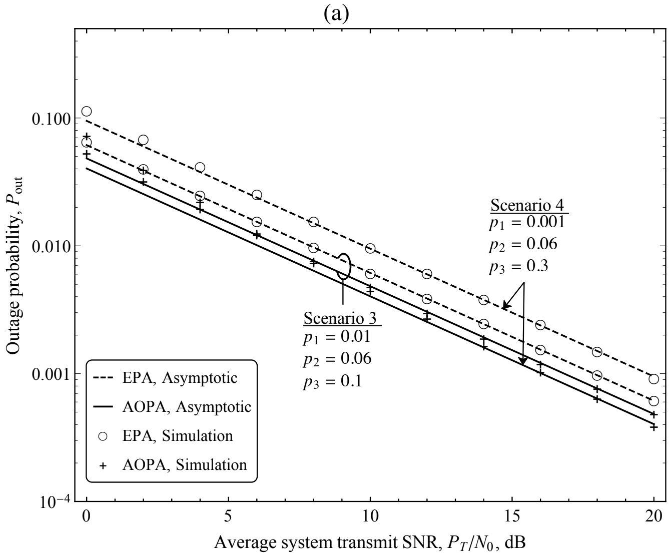
the first relay. The aim is to find out how the channel quality of the first hop impacts the performances of EPA and AOPA. From the curves, we notice that the impact is indeed quite different in each case. Recall that the smaller is p_1, the better is the channel quality of the first hop. For EPA, the smaller is p_1, the higher is the system throughput, but the higher as well is the outage probability. This can be understood as follows. As p_1 decreases, using (17), it can be shown that $I(B_2; B_0|B_1)$ also decreases, whereas $I(B_1; B_0|B_2)$ and $I(B_1, B_2; B_0)$ increase. Accordingly, using (8), the inequality constraint $R_2 \geq I(B_2; B_0|B_1)$ becomes more likely, whereas $R_1 \geq I(B_1; B_0|B_2)$ and $R_1 + R_2 \geq I(B_1, B_2; B_0)$ become less likely. These two trends have opposite effects on the outage probability: the former reduces it, but the latter increases it. From Fig. 5a, it turns out that the overall impact is dominated by the second trend, since the outage probability is observed to increase. As for the throughput, given by (28), it depends on $I(B_1, B_2; B_0)$ and P_{out}. From our previous discussions, as p_1 decreases, both $I(B_1, B_2; B_0)$ and P_{out} increase and, from (28), these two increases prove to have opposite effects on the throughput. From Fig. 5b, it turns out that the overall impact is dominated by $I(B_1, B_2; B_0)$, since the throughput is also observed to increase. In short, for EPA, as a first hop improves (e.g., as p_1 decreases), the throughput improves as well, but the outage probability deteriorates. In contrast, for AOPA, as a first hop improves, both the throughput and the outage probability improve at medium to high SNR, as shown in Figs. 5a and 5b. This is a major advantage of AOPA over EPA. It is achieved by suitably distributing the transmit power among the second hops, in a way that minimizes the outage probability while counteracting any side effects of the first hops.

Fig. 6 displays the impact of the relay position on the outage probability and throughput for two relays and an average system transmit SNR of $P_T/N_0 = 20$ dB. The first relay is fixed at $d_1 = 0.5$, and the second one is located at varying distances d_2 from the destination, ranging from 0 to 1. Three situations are investigated based on scenario 7: (a) identical bit-flipping probabilities ($p_1 = p_2 = 0.001$), (b) smallest bit-flipping probability assigned to the relay with a variable location ($p_1 = 0.1$ and $p_2 = 0.001$), and (c) highest bit-flipping probability assigned to the relay with a variable location ($p_1 = 0.001$ and $p_2 = 0.1$). The outage probability is shown in Fig. 6a and the corresponding AOPA coefficient for the second relay is shown in Fig. 6b. The following can be observed from the curves: (i) in all the cases, our asymptotic outage expression in (26) has an excellent match; (ii) AOPA outperforms EPA, possibly except at a singular relay location, for which AOPA and EPA bear identical performances; (iii) the closer is the relay to the destination, the smaller is the transmit power allocated to it by AOPA. Regarding the observation (ii), the balancing distance d_2 at which EPA and AOPA perform identically is, as expected, $d_2 = d_1 = 0.5$ when the bit-flipping probabilities associated with the two relays are identical. On the other hand, when the bit-flipping probability for the second relay is smaller/higher than that for the first (fixed) relay, the balancing distance d_2 moves toward/outward the destination. There is compensation mechanism at play: if a given relay is moved toward the destination, its second hop tends to improve, but the AOPA criterion counteracts this by reducing the relay's transmit power, keeping an optimal balance between all the second hops. In particular, for scenario (c), note that the outage performances of EPA and AOPA are barely affected by the location of the second relay. This is because in this case the fixed (first) relay has a much better first hop (much smaller bit-flipping probability), dominating the performance.

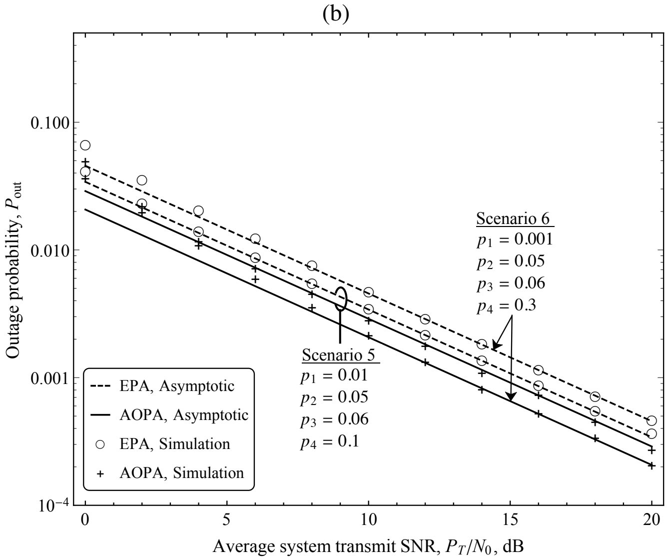
Finally, in Fig. 7, we assess the effectiveness of our AOPA policy when applied to a given practical DSC/JD scheme, namely, that one in [2]. To this end, we have used the same scenarios in Table I, with the following additional simulation parameters: (a) frame length is 10^3 bits; (b) number of frames is 10^5; (c) random interleaving; (d) generator polynomials of SNRCC and SRCC are $G = ([3, 1])_8$ and $G = ([3, 1]2)_8$, respectively; (e) binary phase-shift keying modulation; and (f) doping ratio of ACC is 1. Note the presence of an error floor in all BER curves, as widely known and reported in case of lossy intra-links. From the curves, it can be observed that AOPA outperforms EPA in all the cases, achieving considerable gains for some scenarios. In virtue of the error floor, the SNR gains have been measured, somewhat arbitrarily, at BER $= 0.1$ for scenarios 2, 4, and 6, and at BER $= 0.01$ for scenarios 1, 3, and 5. Those gains are listed in Table I. Note in



(a)

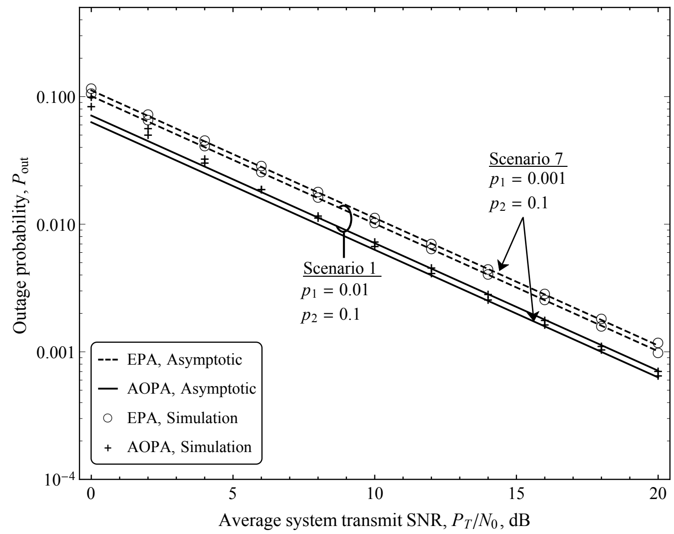


(b)

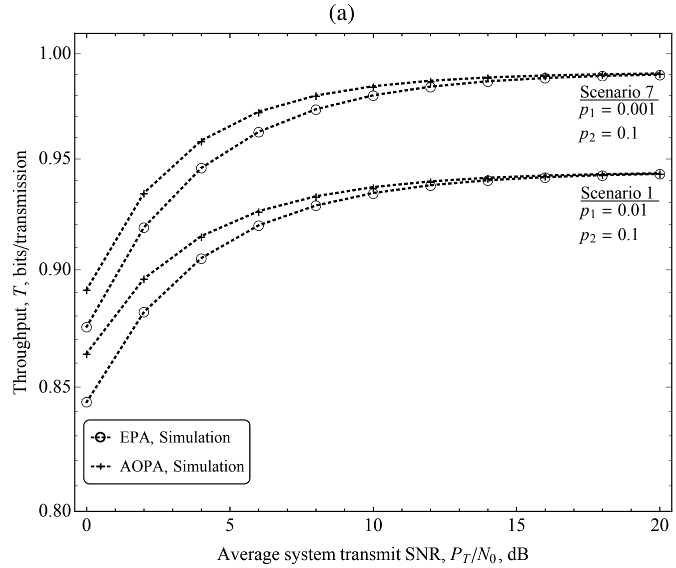


(c)

Fig. 4: Outage comparison between EPA and AOPA under non-identical bit-flipping probabilities: (a) two relays; (b) three relays; (c) four relays. (See Table I for further details.)



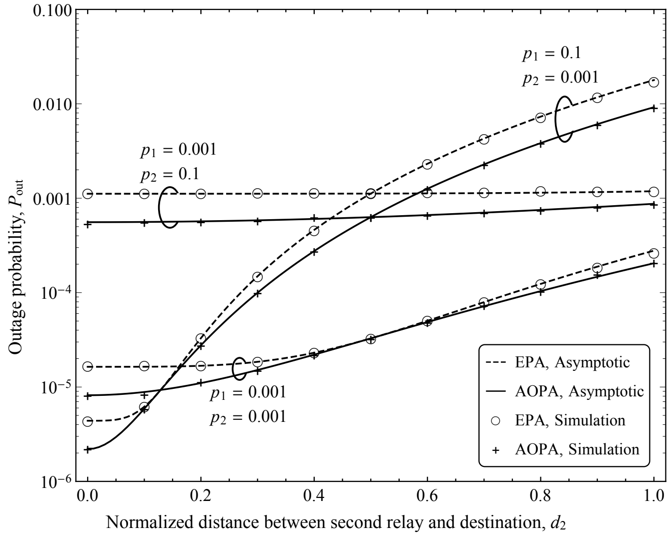
(a)



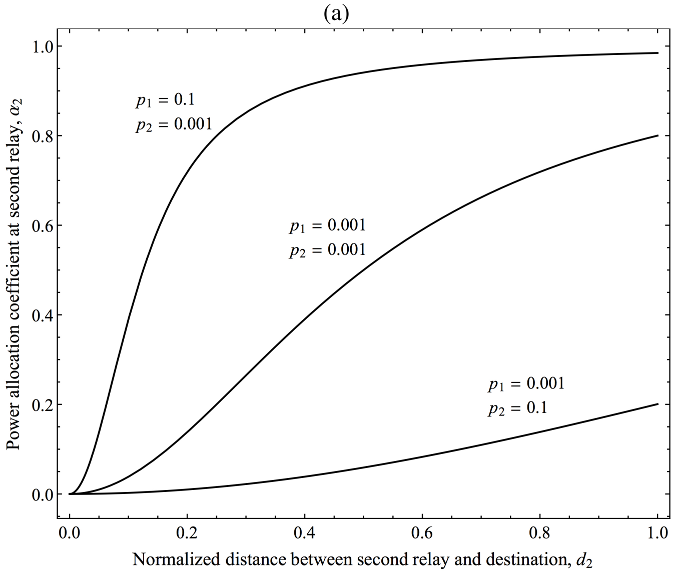
(b)

Fig. 5: Performance comparison between EPA and AOPA for two relays and varying bit-flipping probability at the first relay: (a) outage probability; (b) throughput.

the table how the SNR gains for BER are fully consistent with the corresponding SNR gains for the outage probability. Most importantly, we have empirically verified that the AOPA performance is practically indistinguishable from that of an optimal power allocation scheme, assessed via exhaustive simulation. These results confirm the effectiveness of AOPA for practical coding schemes, which was our ultimate aim. They also confirm that our modified scope of the Slepian-Wolf theorem, introduced here merely as a basis for the power allocation design, is indeed an appropriate framework for this task and thus a good performance indicator of the investigated system.



(a)



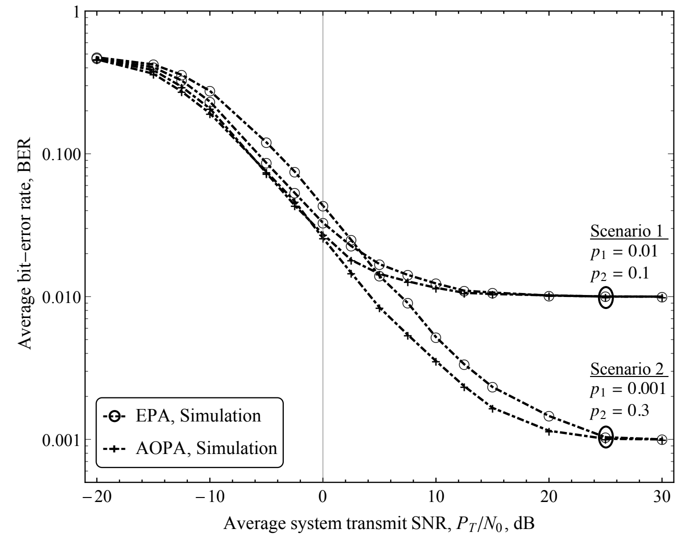
(b)

Fig. 6: Outage comparison between EPA and AOPA for two relays and $P_T/N_0 = 20$ dB in terms of the distance between the second relay and destination: (a) outage probability; (b) AOPA coefficient for the second relay.

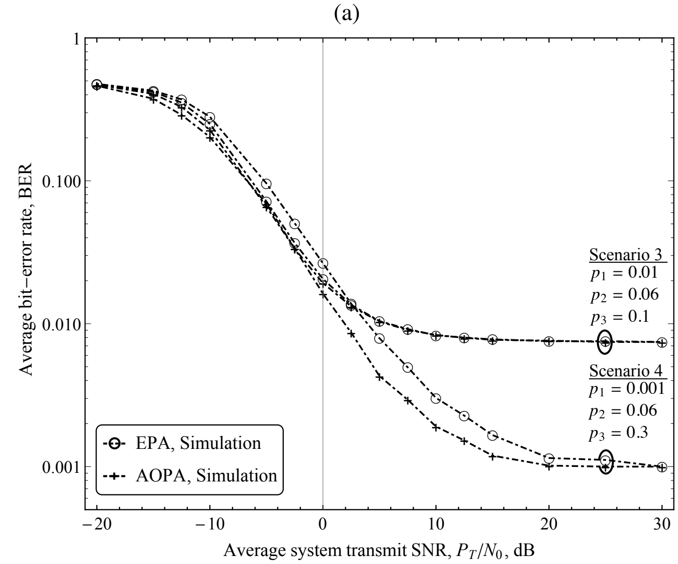
IX. Final Remark

In some scenarios, it may be advantageous to share the second-hop resources among all the relays via an optimized power allocation scheme, but in other scenarios it may be advantageous to use only the best relay for transmission—the relay with the lowest bit-flipping probability at the first hop. For illustration purposes, in the discussion that follows, let us assume a time-division multiple access basis at the second hops, so that the transmission from each relay takes one time slot.

First, consider scenarios 2, 4, and 6 in Fig. 7, for two, three, and four relays, respectively. In all these scenarios, the best relay has a much lower bit-flipping probability than



(a)



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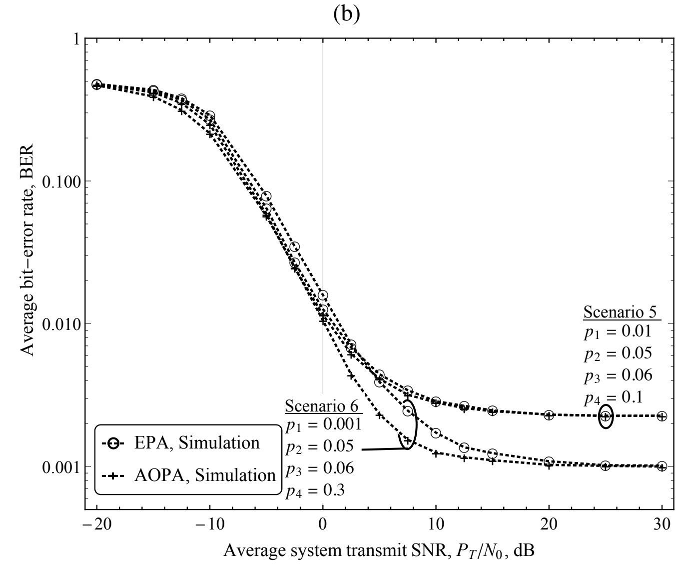

(c)

Fig. 7: Bit-error rate comparison between EPA and AOPA: (a) two relays; (b) three relays; (c) four relays.

the others, namely, $p_1 = 0.001$. As a result, the observed error floor is virtually identical to the bit-flipping probability of the best relay, i.e., around BER $\lesssim 0.001$, regardless of the number N of relays. In such cases, it turns out that a more efficient, alternative scheme—using the same amount of total transmit power (P_T) and the same number of time slots (N)—is to employ only the best relay for transmission. More specifically, the best relay retransmits its message N times to the destination, with transmit power P_T/N, while the other relays remain silent. The destination then combines these replicas, thereby achieving a higher diversity order (the error rate decreases faster as the SNR increases) when compared with the multirelay transmission using an optimized power allocation, while achieving nearly the same error floor (BER $= 0.001$, in the examples). We have confirmed this via simulation results, omitted here for simplicity.

Now, consider scenarios 1, 3, and 5 in Fig. 7, again for two, three, and four relays, respectively. In all these scenarios, the best relay has a bit-flipping probability that is less disparate from the others, namely, $p_1 = 0.01$. In other words, in comparison to scenarios 2, 4, and 6, the corrupted replicas at the various relays contain more similar amounts of information about the source message. As a result, as the number of relays increases, the observed error floor falls significantly below the bit-flipping probability of the best relay, achieving around BER ≈ 0.002 for scenario 5. In contrast, despite its improved diversity order, the alternative scheme using retransmissions exclusively from the best relay is clearly unable to reduce the error floor below the bit-flipping probability assigned to that relay. Therefore, in such cases, the multirelay transmission is advantageous. Indeed, exploiting the cooperative diversity by sharing the total transmit power among various relay routes is the fundamental principle of communication schemes based on the CEO problem.

From the above, it is clear that an overall recommendation on the use of one of, a subset of, or all the available relays depends ultimately on the relative quality of the multiple first hops, represented here by the corresponding bit-flipping probabilities. As a general rule, the more comparable are the bit-flipping probabilities assigned to the various relays, the more beneficial is the joint use of all of them. In any case, with a given set of relays having been chosen for transmission, the power allocation scheme provided in this work gives a nearly optimum performance.

X. Conclusions

In this work, we analyzed a certain outage performance of a general distributed source coding scheme for a multirelay system with intra-link errors and no direct path available from source to destination. More significantly, we capitalized on this outage analysis to design a simple and highly effective power allocation policy for the investigated system. The proposed power allocation was tested into a practical coding scheme. Strikingly, in all the tests, the resulting error-rate performance was nearly optimal. Our results and discussions find important application to emerging links-on-the-fly technologies for robust and efficient communications in unpredictable environments.

Acknowledgements

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Appendix A
Exact and High-SNR Expressions for $J_{2,1}$

The SNRs Γ_1 and Γ_2 are mutually independent, with marginal PDFs given in (2). Using this into (14), and exact closed-form expression for $J_{2,1}$ is obtained as

$$\begin{aligned} J_{2,1} =& 1 - \int_{2^{R_{c1} I(B_1;B_0|B_2)}-1}^{\infty} \int_{2^{R_{c2} I(B_2;B_0|B_1)}-1}^{\infty} \\ & \frac{1}{\bar{\Gamma}_1} e^{-\frac{\gamma_1}{\bar{\Gamma}_1}} \frac{1}{\bar{\Gamma}_2} e^{-\frac{\gamma_2}{\bar{\Gamma}_2}} d\gamma_2 d\gamma_1, \\ =& 1 - \exp\left(-\frac{2^{R_{c1} I(B_1;B_0|B_2)} - 1}{\bar{\Gamma}_1} - \frac{2^{R_{c2} I(B_2;B_0|B_1)} - 1}{\bar{\Gamma}_2}\right). \end{aligned} \tag{37}$$

Assuming $R_{c1} = R_{c2} = R_c$, a corresponding high-SNR expression can be obtained by invoking the approximation $\exp(x) \approx 1 - x$, $x \ll 1$, which gives

$$J_{2,1} \simeq \frac{2^{R_c I(B_1;B_0|B_2)} - 1}{\bar{\Gamma}_1} + \frac{2^{R_c I(B_2;B_0|B_2)} - 1}{\bar{\Gamma}_2}. \tag{38}$$

Appendix B
Exact and High-SNR Expressions for $J_{2,2}$

The SNRs Γ_1 and Γ_2 are mutually independent, with marginal PDFs given in (2). Using this into (15), and exact closed-form expression for $J_{2,1}$ is obtained as

$$\begin{aligned} J_{2,2} =& \int_{2^{R_{c1} I(B_1;B_0|B_2)}-1}^{2^{R_{c1} I(B_1;B_0)}-1} \int_{2^{R_{c2} I(B_2;B_0|B_1)}-1}^{2^{R_{c2} I(B_1,B_2;B_0) - \frac{R_{c2}}{R_{c1}} \log_2(1+\gamma_1)}-1} \\ & \frac{1}{\bar{\Gamma}_1} e^{-\frac{\gamma_1}{\bar{\Gamma}_1}} \frac{1}{\bar{\Gamma}_2} e^{-\frac{\gamma_2}{\bar{\Gamma}_2}} d\gamma_2 d\gamma_1, \\ =& \frac{1}{\bar{\Gamma}_1} \exp\left(-\frac{2^{R_{c2} I(B_2;B_0|B_1)} - 1}{\bar{\Gamma}_2}\right) \int_{2^{R_{c1} I(B_1;B_0|B_2)}-1}^{2^{R_{c1} I(B_1;B_0)}-1} e^{-\frac{\gamma_1}{\bar{\Gamma}_1}} \\ & \left[1 - \exp\left(\frac{2^{R_{c2} I(B_1,B_2;B_0) - \frac{R_{c2}}{R_{c1}} \log_2(1+\gamma_1)}}{\bar{\Gamma}_2} + \right.\right. \\ & \left.\left. \frac{2^{R_{c2} I(B_2;B_0|B_1)}}{\bar{\Gamma}_2}\right)\right] d\gamma_1. \end{aligned} \tag{39}$$

Assuming $R_{c1} = R_{c2} = R_c$, a corresponding high-SNR expression can be obtained by invoking the approximation $\exp(x) \approx 1 - x$, $x \ll 1$, which gives

$$\begin{aligned} J_{2,2} \simeq& \frac{1}{\bar{\Gamma}_1 \bar{\Gamma}_2} \left\{ 2^{R_c I(B_1;B_0|B_2) + R_c I(B_2;B_0|B_1)} \right. \\ & - 2^{R_c I(B_1;B_0) + R_c I(B_2;B_0|B_1)} + 2^{R_c I(B_1,B_2;B_0)} \\ & \left. \times \left[\mathrm{Ei}\left(-\frac{2^{R_c I(B_1;B_0)}}{\bar{\Gamma}_1}\right) - \mathrm{Ei}\left(-\frac{2^{R_c I(B_1;B_0|B_2)}}{\bar{\Gamma}_1}\right) \right] \right\}, \end{aligned} \tag{40}$$

where $\mathrm{Ei}(\cdot)$ is the exponential integral function.

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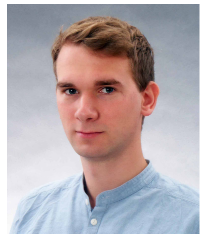

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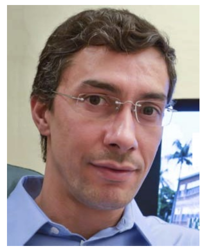

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